SL Paper 1

Let $\sin\theta = \frac{2}{\sqrt{13}}$, where $\frac{\pi}{2} < \theta < \pi$.

a. Find $\cos\theta$. [3]

b. Find $\tan 2\theta$. [5]

Markscheme

a. **METHOD 1**

evidence of choosing $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

correct working ***(A1)***

e.g. $\cos^2\theta = \frac{9}{13}$, $\cos\theta = \pm\frac{3}{\sqrt{13}}$, $\cos\theta = \sqrt{\frac{9}{13}}$

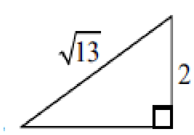
$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

METHOD 2

approach involving Pythagoras' theorem ***(M1)***

e.g. $2^2 + x^2 = 13$,



finding third side equals 3 ***(A1)***

$\cos\theta = -\frac{3}{\sqrt{13}}$ ***A1 N2***

Note: If no working shown, award ***N1*** for $\frac{3}{\sqrt{13}}$.

[3 marks]

b. correct substitution into $\sin 2\theta$ (seen anywhere) ***(A1)***

e.g. $2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)$

correct substitution into $\cos 2\theta$ (seen anywhere) ***(A1)***

e.g. $\left(-\frac{3}{\sqrt{13}}\right)^2 - \left(\frac{2}{\sqrt{13}}\right)^2$, $2\left(-\frac{3}{\sqrt{13}}\right)^2 - 1$, $1 - 2\left(\frac{2}{\sqrt{13}}\right)^2$

valid attempt to find $\tan 2\theta$ ***(M1)***

e.g. $\frac{2\left(\frac{2}{\sqrt{13}}\right)\left(-\frac{3}{\sqrt{13}}\right)}{\left(-\frac{3}{\sqrt{13}}\right)^2-\left(\frac{2}{\sqrt{13}}\right)^2}$, $\frac{2\left(-\frac{2}{3}\right)}{1-\left(-\frac{2}{3}\right)^2}$

correct working ***A1***

e.g. $\frac{\frac{(2)(2)(-3)}{13}}{\frac{9}{13}-\frac{4}{13}}$, $\frac{-\frac{12}{\left(\sqrt{13}\right)^2}}{\frac{18}{13}-1}$, $\frac{-\frac{12}{13}}{\frac{5}{13}}$

$\tan 2\theta = -\frac{12}{5}$ ***A1*** ***N4***

Note: If students find answers for $\cos\theta$ which are not in the range $[-1, 1]$, award full ***FT*** in (b) for correct ***FT*** working shown.

[5 marks]

Examiners report

a. While the majority of candidates knew to use the Pythagorean identity in part (a), very few remembered that the cosine of an angle in the second quadrant will have a negative value.

b. In part (b), many candidates incorrectly tried to calculate $\tan 2\theta$ as $2 \times \tan\theta$, rather than using the double-angle identities.

Let $f(x) = \frac{\cos x}{\sin x}$, for $\sin x \neq 0$.

In the following table, $f'\left(\frac{\pi}{2}\right) = p$ and $f''\left(\frac{\pi}{2}\right) = q$. The table also gives approximate values of $f'(x)$ and $f''(x)$ near $x = \frac{\pi}{2}$.

x	$\frac{\pi}{2}-0.1$	$\frac{\pi}{2}$	$\frac{\pi}{2}+0.1$
$f'(x)$	−1.01	p	−1.01
$f''(x)$	0.203	q	−0.203

a. Use the quotient rule to show that $f'(x) = \frac{-1}{\sin^2 x}$. [5]

b. Find $f''(x)$. [3]

c. Find the value of p and of q. [3]

d. Use information from the table to explain why there is a point of inflexion on the graph of f where $x = \frac{\pi}{2}$. [2]

Markscheme

a. $\frac{d}{dx}\sin x = \cos x$, $\frac{d}{dx}\cos x = -\sin x$ (seen anywhere) ***(A1)(A1)***

evidence of using the quotient rule ***M1***

correct substitution ***A1***

e.g. $\frac{\sin x(-\sin x)-\cos x(\cos x)}{\sin^2 x}$, $\frac{-\sin^2 x-\cos^2 x}{\sin^2 x}$

$f'(x) = \frac{-(\sin^2 x+\cos^2 x)}{\sin^2 x}$ ***A1***

$f'(x) = \frac{-1}{\sin^2 x}$ ***AG*** ***N0***

[5 marks]

b. **METHOD 1**

appropriate approach ***(M1)***

e.g. $f'(x) = -(\sin x)^{-2}$

$f''(x) = 2(\sin^{-3} x)(\cos x) \left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1A1*** ***N3***

Note: Award ***A1*** for $2\sin^{-3} x$, ***A1*** for $\cos x$.

METHOD 2

derivative of $\sin^2 x = 2\sin x \cos x$ (seen anywhere) ***A1***

evidence of choosing quotient rule ***(M1)***

e.g. $u = -1$, $v = \sin^2 x$, $f'' = \frac{\sin^2 x \times 0 - (-1)2\sin x \cos x}{(\sin^2 x)^2}$

$f''(x) = \frac{2\sin x \cos x}{(\sin^2 x)^2} \left(= \frac{2\cos x}{\sin^3 x}\right)$ ***A1*** ***N3***

[3 marks]

c. evidence of substituting $\frac{\pi}{2}$ ***M1***

e.g. $\frac{-1}{\sin^2 \frac{\pi}{2}}$, $\frac{2\cos \frac{\pi}{2}}{\sin^3 \frac{\pi}{2}}$

$p = -1$, $q = 0$ ***A1A1*** ***N1N1***

[3 marks]

d. second derivative is zero, second derivative changes sign ***R1R1*** ***N2***

[2 marks]

Examiners report

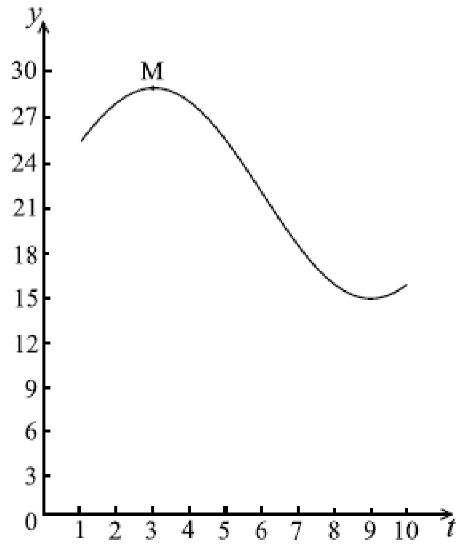
a. Many candidates comfortably applied the quotient rule, although some did not completely show that the Pythagorean identity achieves the numerator of the answer given. Whether changing to $-(\sin x)^{-2}$, or applying the quotient rule a second time, most candidates neglected the chain rule in finding the second derivative.

b. Whether changing to $-(\sin x)^{-2}$, or applying the quotient rule a second time, most candidates neglected the chain rule in finding the second derivative.

c. Those who knew the trigonometric ratios at $\frac{\pi}{2}$typically found the values of p and of q, sometimes in follow-through from an incorrect $f''(x)$.

d. Few candidates gave two reasons from the table that supported the existence of a point of inflexion. Most stated that the second derivative is zero and neglected to consider the sign change to the left and right of q. Some discussed a change of concavity, but without supporting this statement by referencing the change of sign in $f''(x)$, so no marks were earned.

Let $f(t) = a\cos b(t-c) + d$, $t \geq 0$. Part of the graph of $y = f(t)$ is given below.



When $t = 3$, there is a maximum value of 29, at M.

When $t = 9$, there is a minimum value of 15.

a(i), (ii), (iii) and (iv). (i) Find the value of a. [7]

(ii) Show that $b = \frac{\pi}{6}$.

(iii) Find the value of d.

(iv) Write down a value for c.

b. The transformation P is given by a horizontal stretch of a scale factor of $\frac{1}{2}$, followed by a translation of $\begin{pmatrix} 3 \\ -10 \end{pmatrix}$. [2]

Let M' be the image of M under P. Find the coordinates of M' .

c. The graph of g is the image of the graph of f under P. [4]

Find $g(t)$ in the form $g(t) = 7\cos B(t - c) + D$.

d. The graph of g is the image of the graph of f under P. [3]

Give a full geometric description of the transformation that maps the graph of g to the graph of f .

Markscheme

a(i), (ii), (iii) and (iv). (i) attempt to substitute ***(M1)***

e.g. $a = \frac{29-15}{2}$

$a = 7$ (accept $a = -7$) ***A1*** ***N2***

(ii) period $= 12$ ***(A1)***

$b = \frac{2\pi}{12}$ ***A1***

$b = \frac{\pi}{6}$ ***AG*** ***N0***

(iii) attempt to substitute ***(M1)***

e.g. $d = \frac{29+15}{2}$

$d = 22$ ***A1*** ***N2***

(iv) $c = 3$ (accept $c = 9$ from $a = -7$) ***A1*** ***N1***

Note: Other correct values for c can be found, $c = 3 \pm 12k$, $k \in \mathbb{Z}$.

[7 marks]

b. stretch takes 3 to 1.5 ***(A1)***

translation maps $(1.5, 29)$ to $(4.5, 19)$ (so M' is $(4.5, 19)$) ***A1*** ***N2***

[2 marks]

c. $g(t) = 7\cos\frac{\pi}{3}(t - 4.5) + 12$ ***A1A2A1*** ***N4***

Note: Award ***A1*** for $\frac{\pi}{3}$, ***A2*** for 4.5, ***A1*** for 12.

Other correct values for c can be found, $c = 4.5 \pm 6k$, $k \in \mathbb{Z}$.

[4 marks]

d. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ ***(A1)***

horizontal stretch of a scale factor of 2 ***(A1)***

completely correct description, in correct order ***A1*** ***N3***

e.g. translation $\begin{pmatrix} -3 \\ 10 \end{pmatrix}$ then horizontal stretch of a scale factor of 2

[3 marks]

Examiners report

a(i), (ii), (iii) and (iv). This question was the most difficult on the paper. Where candidates attempted this question, part (a) was answered satisfactorily.

b. Few answered part (b) correctly as most could not interpret the horizontal stretch.

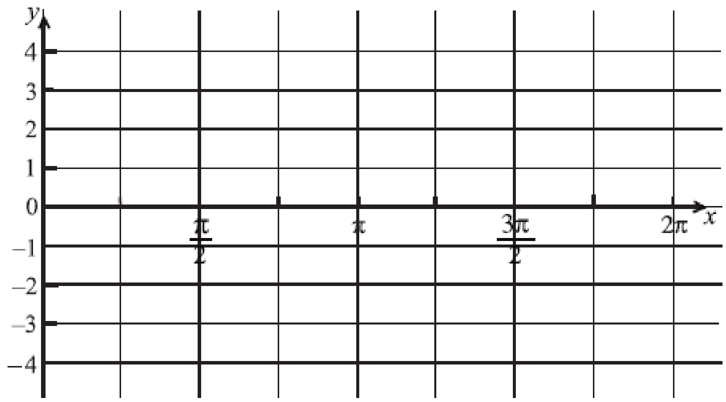
c. Few answered part (b) correctly as most could not interpret the horizontal stretch. As a result, there were many who were unable to answer part (c) although follow through marks were often obtained from incorrect answers in both parts (a) and (b). The link between the answer in (b) and the value of C in part (c) was lost on all but the most attentive.

d. In part (d), some candidates could name the transformations required, although only a handful provided the correct order of the transformations to return the graph to its original state.

Consider $g(x) = 3\sin 2x$.

a. Write down the period of g. [1]

b. On the diagram below, sketch the curve of g, for $0 \le x \le 2\pi$. [3]

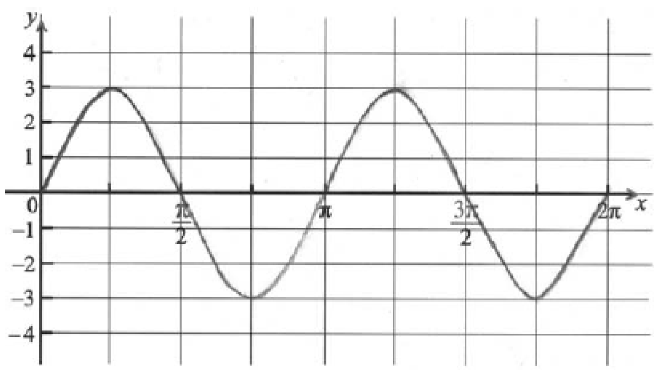


c. Write down the number of solutions to the equation $g(x) = 2$, for $0 \leq x \leq 2\pi$. [2]

Markscheme

a. period $= \pi$ ***A1 N1***

[1 mark]

b. 

A1A1A1 N3

Note: Award ***A1*** for amplitude of 3, ***A1*** for **their** period, ***A1*** for a sine curve passing through $(0, 0)$ and $(0, 2\pi)$.

[3 marks]

c. evidence of appropriate approach ***(M1)***

e.g. line $y = 2$ on graph, discussion of number of solutions in the domain

4 (solutions) ***A1 N2***

[2 marks]

Examiners report

a. Many candidates were unable to write down the period of the function.

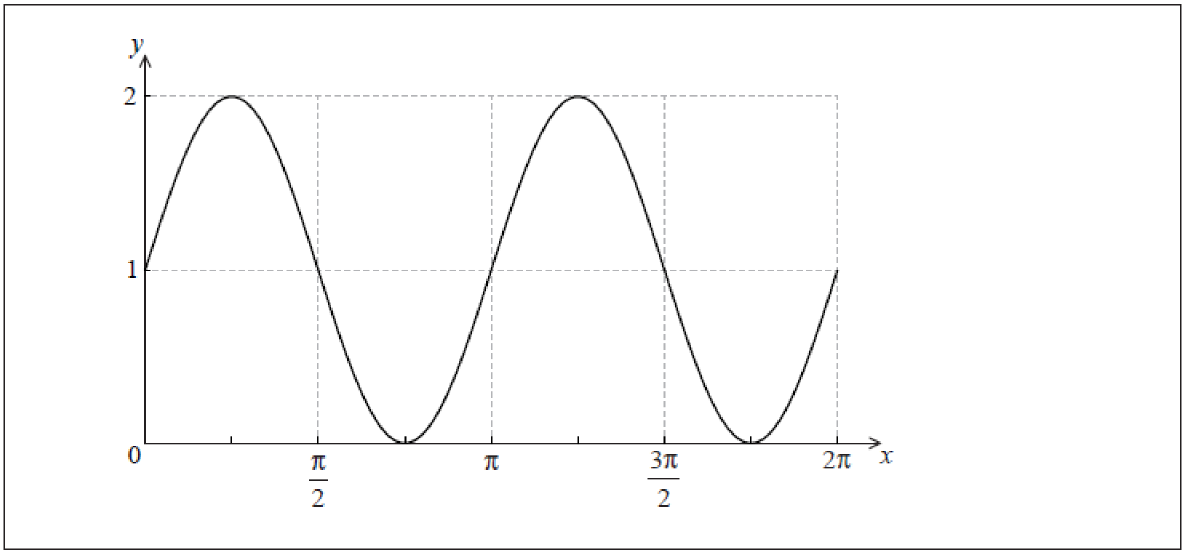
b. Many candidates were unable to write down the period of the function. However, they were often then able to go and correctly sketch the graph with the correct period.

c. The final part was poorly done with many candidates finding the number of zeros instead of the intersection with the line $y = 2$.

Let $f(x) = (\sin x + \cos x)^2$.

a. Show that $f(x)$ can be expressed as $1 + \sin 2x$. [2]

b. The graph of *f* is shown below for $0 \le x \le 2\pi$. [2]



Let $g(x) = 1 + \cos x$. On the same set of axes, sketch the graph of g for $0 \le x \le 2\pi$.

c. The graph of g can be obtained from the graph of *f* under a horizontal stretch of scale factor p followed by a translation by the vector $\begin{pmatrix} k \\ 0 \end{pmatrix}$. [2]

Write down the value of p and a possible value of k .

Markscheme

a. attempt to expand ***(M1)***

e.g. $(\sin x + \cos x)(\sin x + \cos x)$; at least 3 terms

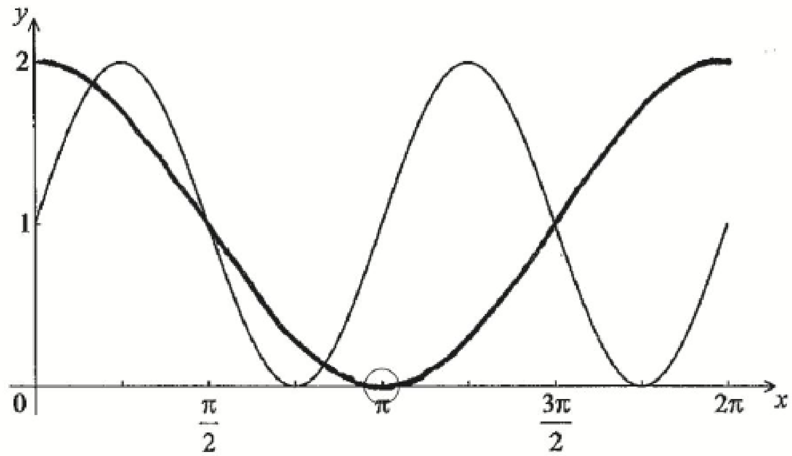
correct expansion ***A1***

e.g. $\sin^2 x + 2\sin x\cos x + \cos^2 x$

$f(x) = 1 + \sin 2x$ ***AG*** ***N0***

[2 marks]

b.



A1A1 *N2*

Note: Award ***A1*** for correct sinusoidal shape with period 2π and range $[0,\,2]$, ***A1*** for minimum in circle.

c. $p = 2$, $k = -\frac{\pi}{2}$ ***A1A1*** ***N2***

[2 marks]

Examiners report

a. Simplifying a trigonometric expression and applying identities was generally well answered in part (a), although some candidates were certainly helped by the fact that it was a "show that" question.

b. More candidates had difficulty with part (b) with many assuming the first graph was $1 + \sin(x)$ and hence sketching a horizontal translation of $\pi/2$ for the graph of *g*; some attempts were not even sinusoidal. While some candidates found the stretch factor *p* correctly or from follow-through on their own graph, very few successfully found the value and direction for the translation.

c. Part (c) certainly served as a discriminator between the grade 6 and 7 candidates.

Let $f(x) = \sqrt{3}\mathrm{e}^{2x}\sin x + \mathrm{e}^{2x}\cos x$, for $0 \le x \le \pi$. Solve the equation $f(x) = 0$.

Markscheme

$\mathrm{e}^{2x}\left(\sqrt{3}\sin x + \cos x\right) = 0$ ***(A1)***

$\mathrm{e}^{2x} = 0$ not possible (seen anywhere) ***(A1)***

simplifying

e.g. $\sqrt{3}\sin x + \cos x = 0$, $\sqrt{3}\sin x = -\cos x$, $\frac{\sin x}{-\cos x} = \frac{1}{\sqrt{3}}$ ***A1***

EITHER

$\tan x = -\frac{1}{\sqrt{3}}$ ***A1***

$x = \frac{5\pi}{6}$ ***A2*** ***N4***

OR

sketch of 30° , 60° , 90° triangle with sides 1, 2, $\sqrt{3}$ ***A1*** work leading to $x = \frac{5\pi}{6}$ ***A1***

verifying $\frac{5\pi}{6}$ satisfies equation ***A1*** ***N4***

[6 marks]

Examiners report

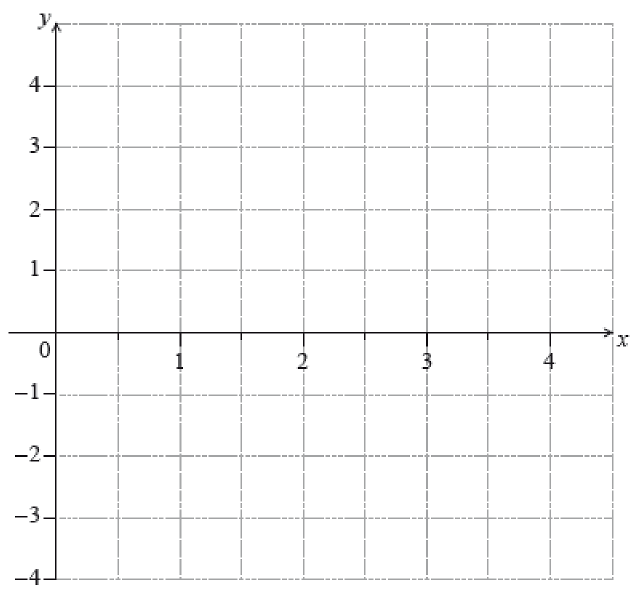
Those who realized e^{2x} was a common factor usually earned the first four marks. Few could reason with the given information to solve the equation from there. There were many candidates who attempted some fruitless algebra that did not include factorization.

Let $f(x) = 3\sin\left(\frac{\pi}{2}x\right)$, for $0 \leqslant x \leqslant 4$.

a. (i) Write down the amplitude of f. [3]

(ii) Find the period of f.

b. On the following grid sketch the graph of f. [4]



Markscheme

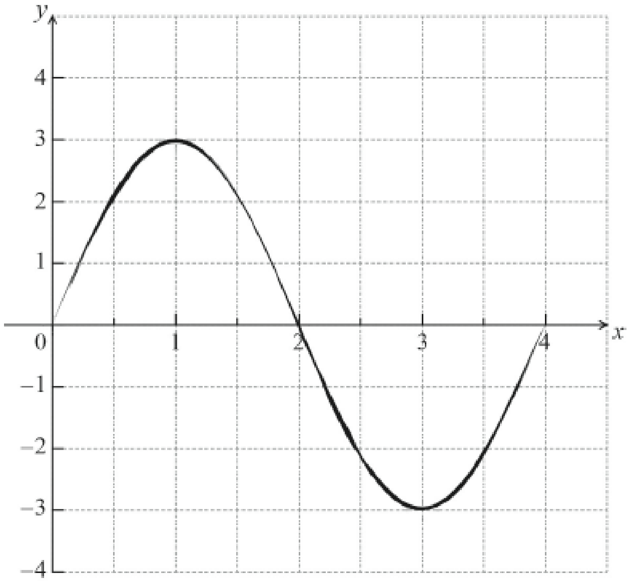
a. (i) 3 ***A1*** ***N1***

(ii) valid attempt to find the period ***(M1)***

eg $\frac{2\pi}{b}$, $\frac{2\pi}{\frac{\pi}{2}}$

period $= 4$ ***A1*** ***N2***

[3 marks]

b. 

A1A1A1A1 *N4*

[4 marks]

Examiners report

a. Almost all candidates correctly stated the amplitude but then had difficulty finding the correct period. Few students faced problems in sketching the graph of the given function, even if they had found the wrong period, thus indicating a lack of understanding of the term ‘period’ in part a(ii). Most sketches were good although care should be taken to observe the given domain and to draw a neat curve.

b. Almost all candidates correctly stated the amplitude but then had difficulty finding the correct period. Few students faced problems in sketching the graph of the given function, even if they had found the wrong period, thus indicating a lack of understanding of the term ‘period’ in part a(ii). Most sketches were good although care should be taken to observe the given domain and to draw a neat curve.

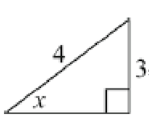
Given that $\sin x = \frac{3}{4}$, where x is an obtuse angle,

a. find the value of $\cos x$; [4]

b. find the value of $\cos 2x$. [3]

Markscheme

a. valid approach ***(M1)***

eg 

, $\sin^2 x + \cos^2 x = 1$

correct working ***(A1)***

eg $4^2 - 3^2,\ \cos^2 x = 1 - \left(\frac{3}{4}\right)^2$

correct calculation ***(A1)***

eg $\frac{\sqrt{7}}{4},\ \cos^2 x = \frac{7}{16}$

$\cos x = -\frac{\sqrt{7}}{4}$ ***A1 N3***

[4 marks]

b. correct substitution (accept missing minus with cos) ***(A1)***

eg $1-2\left(\frac{3}{4}\right)^2,\ 2\left(-\frac{\sqrt{7}}{4}\right)^2-1,\ \left(\frac{\sqrt{7}}{4}\right)^2-\left(\frac{3}{4}\right)^2$

correct working ***A1***

eg $2\left(\frac{7}{16}\right)-1,\ 1-\frac{18}{16},\ \frac{7}{16}-\frac{9}{16}$

$\cos 2x = -\frac{2}{16}\ \left(=-\frac{1}{8}\right)$ ***A1 N2***

[3 marks]

Total [7 marks]

Examiners report

a. Many candidates were able to find the cosine ratio of $\frac{\sqrt{7}}{4}$ but did not take into account the information about the obtuse angle and seldom selected the negative answer. Finding $\cos 2x$ proved easier; the most common error seen was $\cos 2x = 2\cos x$.

b. Many candidates were able to find the cosine ratio of $\frac{\sqrt{7}}{4}$ but did not take into account the information about the obtuse angle and seldom selected the negative answer. Finding $\cos 2x$ proved easier; the most common error seen was $\cos 2x = 2\cos x$.

Note: In this question, distance is in metres and time is in seconds.

Two particles P_1 and P_2 start moving from a point A at the same time, along different straight lines.

After t seconds, the position of P_1 is given by ***r*** = $\begin{pmatrix}4\\-1\\3\end{pmatrix}+t\begin{pmatrix}1\\2\\-1\end{pmatrix}$.

Two seconds after leaving A, P_1 is at point B.

Two seconds after leaving A, P_2 is at point C, where $\overrightarrow{\mathrm{AC}}=\begin{pmatrix}3\\0\\4\end{pmatrix}$.

a. Find the coordinates of A. [2]

b.i. Find $\overrightarrow{\mathrm{AB}}$; [3]

b.ii. Find $\left|\overrightarrow{\mathrm{AB}}\right|$. [2]

c. Find $\cos \mathrm{B\hat{A}C}$. [5]

d. Hence or otherwise, find the distance between P_1 and P_2 two seconds after they leave A. [4]

Markscheme

a. recognizing $t = 0$ at A ***(M1)***

A is $(4,\ -1,\ 3)$ ***A1 N2***

[2 marks]

b.i. **METHOD 1**

valid approach ***(M1)***

eg $\begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} + 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},\ (6,\ 3,\ -1)$

correct approach to find $\overrightarrow{\mathrm{AB}}$ ***(A1)***

eg $\mathrm{AO} + \mathrm{OB},\ \mathrm{B} - \mathrm{A},\ \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

METHOD 2

recognizing $\overrightarrow{\mathrm{AB}}$ is two times the direction vector ***(M1)***

correct working ***(A1)***

eg $\overrightarrow{\mathrm{AB}} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix}$ ***A1 N2***

[3 marks]

b.ii. correct substitution ***(A1)***

eg $\left|\overrightarrow{\mathrm{AB}}\right| = \sqrt{2^2 + 4^2 + 4^2},\ \sqrt{4 + 16 + 16},\ \sqrt{36}$

$\left|\overrightarrow{\mathrm{AB}}\right| = 6$ ***A1 N2***

[2 marks]

c. **METHOD 1 (vector approach)**

valid approach involving $\overrightarrow{\mathrm{AB}}$ and $\overrightarrow{\mathrm{AC}}$ ***(M1)***

eg $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}},\ \frac{\overrightarrow{\mathrm{BA}} \bullet \overrightarrow{\mathrm{AC}}}{\mathrm{AB} \times \mathrm{AC}}$

finding scalar product and $\left|\overrightarrow{\mathrm{AC}}\right|$ ***(A1)(A1)***

scalar product $2(3) + 4(0) - 4(4)\ (= -10)$

$\left|\overrightarrow{\mathrm{AC}}\right| = \sqrt{3^2 + 0^2 + 4^2}\ (= 5)$

substitution of **their** scalar product and magnitudes into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{6 + 0 - 16}{6\sqrt{3^2 + 4^2}}$

$\cos B\hat{A}C = -\frac{10}{30}\left(= -\frac{1}{3}\right)$ ***A1 N2***

METHOD 2 (triangle approach)

valid approach involving cosine rule ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{\mathrm{AB}^2+\mathrm{AC}^2-\mathrm{BC}^2}{2\times\mathrm{AB}\times\mathrm{AC}}$

finding lengths AC and BC ***(A1)(A1)***

$\mathrm{AC} = 5,\ \mathrm{BC} = 9$

substitution of **their** lengths into cosine formula ***(M1)***

eg $\cos \mathrm{B\hat{A}C} = \frac{5^2+6^2-9^2}{2\times5\times6}$

$\cos \mathrm{B\hat{A}C} = -\frac{20}{60}\ \left(= -\frac{1}{3}\right)$ ***A1 N2***

[5 marks]

d. **Note:** Award relevant marks for working seen to find BC in part (c) (if cosine rule used in part (c)).

METHOD 1 (using cosine rule)

recognizing need to find BC ***(M1)***

choosing cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - 2ab\cos \mathrm{C}$

correct substitution into RHS ***A1***

eg $\mathrm{BC}^2 = (6)^2 + (5)^2 - 2(6)(5)\left(-\frac{1}{3}\right),\ 36+25+20$

distance is 9 ***A1 N2***

METHOD 2 (finding magnitude of $\overrightarrow{BC}$)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find $\overrightarrow{\mathrm{OB}}$ or $\overrightarrow{\mathrm{OC}}$, $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 6 \\ 3 \\ -1 \end{pmatrix}$ or $\overrightarrow{\mathrm{OC}} = \begin{pmatrix} 7 \\ -1 \\ 7 \end{pmatrix}$, $\overrightarrow{\mathrm{BA}} + \overrightarrow{\mathrm{AC}}$

correct working ***A1***

eg $\overrightarrow{\mathrm{BC}} = \begin{pmatrix} 1 \\ -4 \\ 8 \end{pmatrix},\ \overrightarrow{\mathrm{CB}} = \begin{pmatrix} -1 \\ 4 \\ -8 \end{pmatrix},\ \sqrt{1^2+4^2+8^2} = \sqrt{81}$

distance is 9 ***A1 N2***

METHOD 3 (finding coordinates and using distance formula)

recognizing need to find BC ***(M1)***

valid approach ***(M1)***

eg attempt to find coordinates of B or C, $\mathrm{B}(6,\ 3,\ -1)$ or $\mathrm{C}(7,\ -1,\ 7)$

correct substitution into distance formula ***A1***

eg $\mathrm{BC} = \sqrt{(6-7)^2 + (3-(-1))^2 + (-1-7)^2},\ \sqrt{1^2+4^2+8^2} = \sqrt{81}$

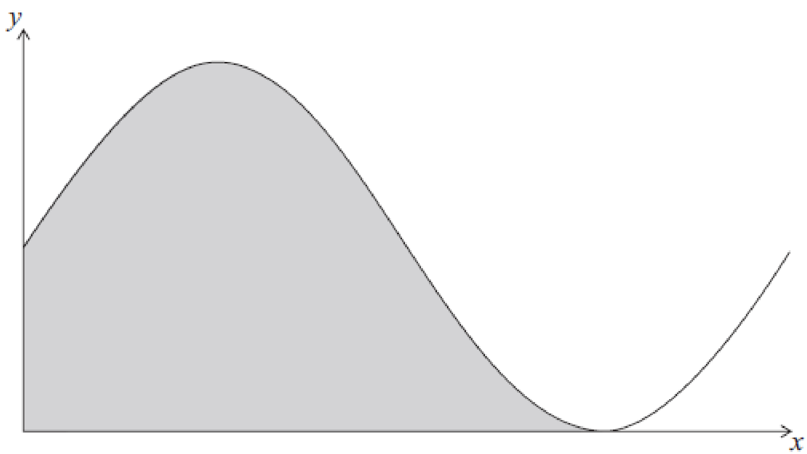
distance is 9 ***A1 N2***

[4 marks]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d. [N/A]

Let $f(x) = 6 + 6\sin x$. Part of the graph of *f* is shown below.



The shaded region is enclosed by the curve of *f* , the *x*-axis, and the *y*-axis.

a(i) and (ii). Solve for $0 \le x < 2\pi$ [5]

(i) $6 + 6\sin x = 6$;

(ii) $6 + 6\sin x = 0$.

b. Write down the exact value of the *x*-intercept of *f* , for $0 \le x < 2\pi$. [1]

c. The area of the shaded region is k . Find the value of k , giving your answer in terms of π . [6]

d. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of *f* is transformed to the graph of *g*. [2]

Give a full geometric description of this transformation.

e. Let $g(x) = 6 + 6\sin\left(x - \frac{\pi}{2}\right)$. The graph of *f* is transformed to the graph of *g*. [3]

Given that $\int_p^{p+\frac{3\pi}{2}} g(x)\mathrm{d}x = k$ and $0 \le p < 2\pi$, write down the two values of *p*.

Markscheme

a(i) and (ii). (i) $\sin x = 0$ ***A1***

$x = 0$, $x = \pi$ ***A1A1*** ***N2***

(ii) $\sin x = -1$ ***A1***

$x = \frac{3\pi}{2}$ ***A1*** ***N1***

[5 marks]

b. $\frac{3\pi}{2}$ ***A1 N1***

[1 mark]

c. evidence of using anti-differentiation ***(M1)***

e.g. $\int_0^{\frac{3\pi}{2}}(6+6\sin x)\mathrm{d}x$

correct integral $6x-6\cos x$ (seen anywhere) ***A1A1***

correct substitution ***(A1)***

e.g. $6\left(\frac{3\pi}{2}\right)-6\cos\left(\frac{3\pi}{2}\right)-(-6\cos 0)$, $9\pi-0+6$

$k=9\pi+6$ ***A1A1 N3***

[6 marks]

d. translation of $\begin{pmatrix}\frac{\pi}{2}\\0\end{pmatrix}$ ***A1A1 N2***

[2 marks]

e. recognizing that the area under *g* is the same as the shaded region in *f* ***(M1)***

$p=\frac{\pi}{2}$, $p=0$ ***A1A1 N3***

[3 marks]

Examiners report

a(i) and (ii). Many candidates again had difficulty finding the common angles in the trigonometric equations. In part (a), some did not show sufficient working in solving the equations. Others obtained a single solution in (a)(i) and did not find another. Some candidates worked in degrees; the majority worked in radians.

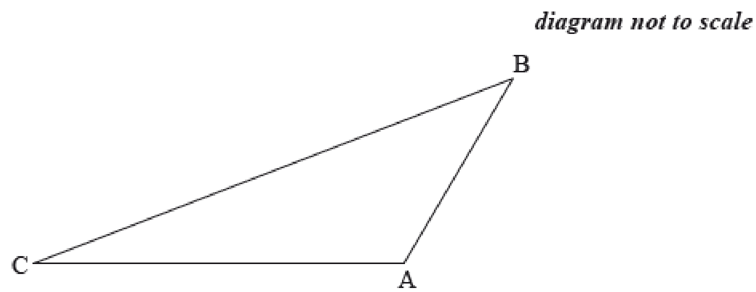
b. While some candidates appeared to use their understanding of the graph of the original function to find the *x*-intercept in part (b), most used their working from part (a)(ii) sometimes with follow-through on an incorrect answer.

c. Most candidates recognized the need for integration in part (c) but far fewer were able to see the solution through correctly to the end. Some did not show the full substitution of the limits, having incorrectly assumed that evaluating the integral at 0 would be 0; without this working, the mark for evaluating at the limits could not be earned. Again, many candidates had trouble working with the common trigonometric values.

d. While there was an issue in the wording of the question with the given domains, this did not appear to bother candidates in part (d). This part was often well completed with candidates using a variety of language to describe the horizontal translation to the right by $\frac{\pi}{2}$.

e. Most candidates who attempted part (e) realized that the integral was equal to the value that they had found in part (c), but a majority tried to integrate the function *g* without success. Some candidates used sketches to find one or both values for *p*. The problem in the wording of the question did not appear to have been noticed by candidates in this part either.

The following diagram shows triangle ABC.



Let $\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}} = -5\sqrt{3}$ and $\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right| = 10$. Find the area of triangle ABC.

Markscheme

attempt to find $\cos \mathrm{C\hat{A}B}$ (seen anywhere) ***(M1)***

eg $\cos\theta = \frac{\overrightarrow{\mathrm{AB}} \bullet \overrightarrow{\mathrm{AC}}}{\left|\overrightarrow{\mathrm{AB}}\right|\left|\overrightarrow{\mathrm{AC}}\right|}$

$\cos \mathrm{C\hat{A}B} = \frac{-5\sqrt{3}}{10} \quad \left(= -\frac{\sqrt{3}}{2}\right)$ ***A1***

valid attempt to find $\sin \mathrm{C\hat{A}B}$ ***(M1)***

eg triangle, Pythagorean identity, $\mathrm{C\hat{A}B} = \frac{5\pi}{6},\ 150^\circ$

$\sin \mathrm{C\hat{A}B} = \frac{1}{2}$ ***(A1)***

correct substitution into formula for area ***(A1)***

eg $\frac{1}{2} \times 10 \times \frac{1}{2},\ \frac{1}{2} \times 10 \times \sin\frac{\pi}{6}$

$\text{area} = \frac{10}{4} \quad \left(= \frac{5}{2}\right)$ ***A1 N3***

[6 marks]

Examiners report

The large majority of candidates were able to find the correct expression for $\cos \mathrm{C\hat{A}B}$, but few recognized that an angle with a negative cosine will be obtuse, rather than acute, and many stated that $\mathrm{C\hat{A}B} = 30^\circ$. When substituting into the triangle area formula, a common error was to substitute $5\sqrt{3}$ rather than 10, as many did not understand the relationship between the magnitude of a vector and the length of a line segment in the triangle formula.

Some of the G2 comments from schools suggested that it might have been easier for their students if this question were split into two parts. While we do tend to provide more support on the earlier questions in the paper, questions 6 and 7 are usually presented with little or no scaffolding. On these later questions, the candidates are often required to use knowledge from different areas of the syllabus within a single question.

Solve $\cos 2x - 3\cos x - 3 - \cos^2 x = \sin^2 x$, for $0 \le x \le 2\pi$.

Markscheme

evidence of substituting for $\cos 2x$ ***(M1)***

evidence of substituting into $\sin^2 x + \cos^2 x = 1$ ***(M1)*** correct equation in terms of $\cos x$ (seen anywhere) ***A1***

e.g. $2\cos^2 x - 1 - 3\cos x - 3 = 1$, $2\cos^2 x - 3\cos x - 5 = 0$

evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

appropriate working ***A1***

e.g. $(2\cos x - 5)(\cos x + 1) = 0$, $(2x - 5)(x + 1)$, $\cos x = \frac{3 \pm \sqrt{49}}{4}$

correct solutions to the equation

e.g. $\cos x = \frac{5}{2}$, $\cos x = -1$, $x = \frac{5}{2}$, $x = -1$ ***(A1)***

$x = \pi$ ***A1*** ***N4***

[7 marks]

Examiners report

This question was quite difficult for most candidates. A number of students earned some credit for manipulating the equation with identities, but many earned no further marks due to algebraic errors. Many did not substitute for $\cos 2x$; others did this substitution but then did nothing further.

Few candidates were able to get a correct equation in terms of $\cos x$ and many who did get the equation didn't know what to do with it. Candidates who correctly solved the resulting quadratic usually found the one correct value of x, earning full marks.

Let $f : x \mapsto \sin^3 x$.

a. (i) Write down the range of the function f. [5]

(ii) Consider $f(x) = 1$, $0 \le x \le 2\pi$. Write down the number of solutions to this equation. Justify your answer.

b. Find $f'(x)$, giving your answer in the form $a\sin^p x\cos^q x$ where $a, p, q \in \mathbb{Z}$. [2]

c. Let $g(x) = \sqrt{3}\sin x(\cos x)^{\frac{1}{2}}$ for $0 \le x \le \frac{\pi}{2}$. Find the volume generated when the curve of g is revolved through 2π about the x-axis. [7]

Markscheme

a. (i) range of f is $[-1, 1]$, $(-1 \le f(x) \le 1)$ ***A2*** ***N2***

(ii) $\sin^3 x \Rightarrow 1 \Rightarrow \sin x = 1$ ***A1***

justification for one solution on $[0, 2\pi]$ ***R1***

e.g. $x = \frac{\pi}{2}$, unit circle, sketch of $\sin x$

1 solution (seen anywhere) ***A1*** ***N1***

[5 marks]

b. $f'(x) = 3\sin^2 x\cos x$ ***A2*** ***N2***

[2 marks]

c. using $V = \int_a^b \pi y^2 \mathrm{d}x$ ***(M1)***

$V = \int_0^{\frac{\pi}{2}} \pi(\sqrt{3}\sin x\cos^{\frac{1}{2}} x)^2 \mathrm{d}x$ ***(A1)***

$= \pi \int_0^{\frac{\pi}{2}} 3\sin^2 x \cos x \mathrm{d}x$ ***A1***

$V = \pi\left[\sin^3 x\right]_0^{\frac{\pi}{2}} \left(= \pi\left(\sin^3\left(\frac{\pi}{2}\right) - \sin^3 0\right)\right)$ ***A2***

evidence of using $\sin\frac{\pi}{2} = 1$ and $\sin 0 = 0$ ***(A1)***

e.g. $\pi(1-0)$

$V = \pi$ ***A1*** ***N1***

[7 marks]

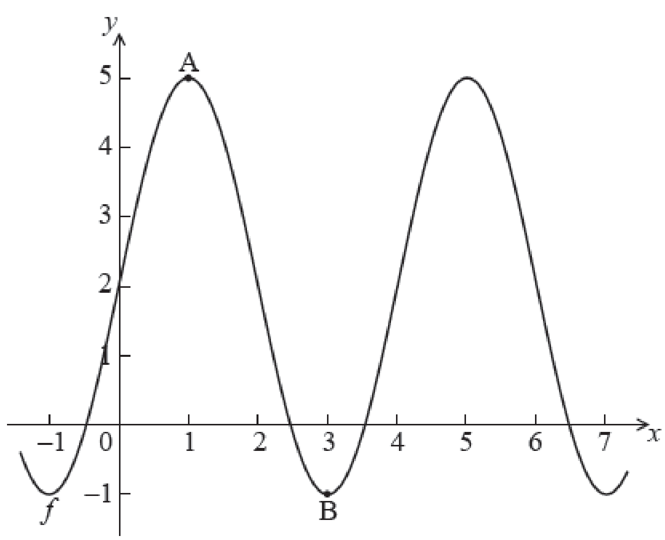
Examiners report

a. This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$.

b. This question was not done well by most candidates.

c. This question was not done well by most candidates. No more than one-third of them could correctly give the range of $f(x) = \sin^3 x$ and few could provide adequate justification for there being exactly one solution to $f(x) = 1$ in the interval $[0, 2\pi]$. Finding the derivative of this function also presented major problems, thus making part (c) of the question much more difficult. In spite of the formula for volume of revolution being given in the Information Booklet, fewer than half of the candidates could correctly put the necessary function and limits into $\pi\int_a^b y^2 \mathrm{d}x$ and fewer still could square $\sqrt{3}\sin x\cos^{\frac{1}{2}} x$ correctly. From those who did square correctly, the correct antiderivative was not often recognized. All manner of antiderivatives were suggested instead.

The diagram below shows part of the graph of a function f .



The graph has a maximum at A$(1, 5)$ and a minimum at B$(3, -1)$.

The function f can be written in the form $f(x) = p\sin(qx) + r$. Find the value of

. (a) p [6]

(b) q

(c) r .

a. p [2]

b. q [2]

c. r . [2]

Markscheme

. (a) valid approach to find p ***(M1)***

eg amplitude $= \frac{\max - \min}{2}$, $p = 6$

$p = 3$ ***A1*** ***N2***

[2 marks]

(b) valid approach to find q ***(M1)***

eg period = 4 , $q = \frac{2\pi}{\text{period}}$

$q = \frac{\pi}{2}$ ***A1*** ***N2***

[2 marks]

(c) valid approach to find r ***(M1)***

eg axis = $\frac{\max + \min}{2}$, sketch of horizontal axis, $f(0)$

$r = 2$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

a. valid approach to find p ***(M1)***

eg amplitude $= \frac{\max - \min}{2}$, $p = 6$

$p = 3$ ***A1*** ***N2***

[2 marks]

b. valid approach to find q ***(M1)***

eg period = 4 , $q = \frac{2\pi}{\text{period}}$

$q = \frac{\pi}{2}$ ***A1*** ***N2***

[2 marks]

c. valid approach to find r ***(M1)***

eg axis = $\frac{\max + \min}{2}$, sketch of horizontal axis, $f(0)$

$r = 2$ ***A1*** ***N2***

[2 marks]

Total [6 marks]

Examiners report

. Many candidates were able to answer all three parts of this question with no difficulty. Some candidates ran into problems when they attempted to substitute into the equation of the function with the parameters p,q and r. The successful candidates were able to find the answers using the given points and their understanding of the different transformations.

Part (b) seemed to be the most difficult, with some candidates not understanding the relationship between q and the period of the function. There were also some candidates who showed working such as $\frac{2\pi}{b}$ without explaining what b represented.

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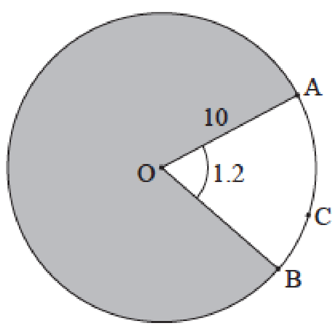
The following diagram shows a circle with centre O and a radius of 10 cm. Points A, B and C lie on the circle.



Angle AOB is 1.2 radians.

a. Find the length of arc ACB. [2]

b. Find the perimeter of the shaded region. [3]

Markscheme

a. correct substitution ***(A1)***

eg $10(1.2)$

ACB is $12\ (\mathrm{cm})$ ***A1 N2***

[2 marks]

b. valid approach to find major arc ***(M1)*** *eg* circumference $-\mathrm{AB}$, major angle $\mathrm{AOB} \times \mathrm{radius}$

correct working for arc length ***(A1)***

eg $2\pi(10) - 12,\ 10(2 \times 3.142 - 1.2),\ 2\pi(10) - 12 + 20$

perimeter is $20\pi + 8 \quad (= 70.8)\ (\mathrm{cm})$ ***A1 N2***

[3 marks]

Total [5 marks]

Examiners report

a. Most candidates were able to find the minor arc length. Similarly most candidates successfully found the major arc length in part b) but did not go on to add the two radii. Quite a few candidates worked with decimal approximations, rather than in terms of π.

b. Most candidates were able to find the minor arc length. Similarly most candidates successfully found the major arc length in part b) but did not go on to add the two radii. Quite a few candidates worked with decimal approximations, rather than in terms of π.

The following table shows the probability distribution of a discrete random variable A, in terms of an angle θ.

a	1	2
$\mathrm{P}(A = a)$	$\cos\theta$	$2\cos 2\theta$

a. Show that $\cos\theta = \frac{3}{4}$. [6]

b. Given that $\tan\theta > 0$, find $\tan\theta$. [3]

c. Let $y = \frac{1}{\cos x}$, for $0 < x < \frac{\pi}{2}$. The graph of $y$between $x = \theta$ and $x = \frac{\pi}{4}$ is rotated 360° about the x-axis. Find the volume of the solid formed. [6]

Markscheme

a. evidence of summing to 1 ***(M1)***

eg $\sum p = 1$

correct equation ***A1***

eg $\cos\theta + 2\cos 2\theta = 1$

correct equation in $\cos\theta$ ***A1***

eg $\cos\theta + 2(2\cos^2\theta - 1) = 1,\ 4\cos^2\theta + \cos\theta - 3 = 0$

evidence of valid approach to solve quadratic ***(M1)***

eg factorizing equation set equal to 0, $\frac{-1 \pm \sqrt{1 - 4 \times 4 \times (-3)}}{8}$

correct working, clearly leading to required answer ***A1***

eg $(4\cos\theta - 3)(\cos\theta + 1),\ \frac{-1 \pm 7}{8}$

correct reason for rejecting $\cos\theta \neq -1$ ***R1***

eg $\cos\theta$ is a probability (value must lie between 0 and 1), $\cos\theta > 0$

Note: Award ***R0*** for $\cos\theta \neq -1$ without a reason.

$\cos\theta = \frac{3}{4}$ ***AG N0***

b. valid approach ***(M1)***

eg sketch of right triangle with sides 3 and 4, $\sin^2 x + \cos^2 x = 1$

correct working

(A1)

eg missing side $= \sqrt{7}$, $\frac{\frac{\sqrt{7}}{4}}{\frac{3}{4}}$

$\tan\theta = \frac{\sqrt{7}}{3}$ ***A1 N2***

[3 marks]

c. attempt to substitute either limits or the function into formula involving f^2 ***(M1)***

eg $\pi\int_\theta^{\frac{\pi}{4}} f^2$, $\int\left(\frac{1}{\cos x}\right)^2$

correct substitution of both limits and function ***(A1)***

eg $\pi\int_\theta^{\frac{\pi}{4}}\left(\frac{1}{\cos x}\right)^2 \mathrm{d}x$

correct integration ***(A1)***

eg $\tan x$

substituting **their** limits into **their** integrated function and subtracting ***(M1)***

eg $\tan\frac{\pi}{4} - \tan\theta$

Note: Award ***M0*** if they substitute into original or differentiated function.

$\tan\frac{\pi}{4} = 1$ ***(A1)***

eg $1 - \tan\theta$

$V = \pi - \frac{\pi\sqrt{7}}{3}$ ***A1 N3***

[6 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

---

a. Show that $4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$. [2]

b. **Hence**, solve the equation $4 - \cos 2\theta + 5\sin\theta = 0$ for $0 \le \theta \le 2\pi$. [5]

Markscheme

a. attempt to substitute $1 - 2\sin^2\theta$ for $\cos 2\theta$ ***(M1)***

correct substitution ***A1***

e.g. $4 - (1 - 2\sin^2\theta) + 5\sin\theta$

$4 - \cos 2\theta + 5\sin\theta = 2\sin^2\theta + 5\sin\theta + 3$ ***AG N0***

[2 marks]

b. evidence of appropriate approach to solve ***(M1)***

e.g. factorizing, quadratic formula

correct working ***A1***

e.g. $(2\sin\theta + 3)(\sin\theta + 1)$, $(2x + 3)(x + 1) = 0$, $\sin x = \frac{-5\pm\sqrt{1}}{4}$

correct solution $\sin\theta = -1$ (do not penalise for including $\sin\theta = -\frac{3}{2}$ ***(A1)***

$\theta = \frac{3\pi}{2}$ ***A2 N3***

[5 marks]

Examiners report

a. In part (a), most candidates successfully substituted using the double-angle formula for cosine. There were quite a few candidates who worked backward, starting with the required answer and manipulating the equation in various ways. As this was a "show that" question, working backward from the given answer is not a valid method.

b. In part (b), many candidates seemed to realize what was required by the word “hence”, though some had trouble factoring the quadratic-type equation. A few candidates were also successful using the quadratic formula. Some candidates got the wrong solution to the equation $\sin\theta = -1$, and there were a few who did not realize that the equation $\sin\theta = -\frac{3}{2}$ has no solution.

Solve $\log_2(2\sin x) + \log_2(\cos x) = -1$, for $2\pi < x < \frac{5\pi}{2}$.

Markscheme

correct application of $\log a + \log b = \log ab$ ***(A1)***

eg $\log_2(2\sin x\cos x),\ \log 2 + \log(\sin x) + \log(\cos x)$

correct equation without logs ***A1***

eg $2\sin x\cos x = 2^{-1},\ \sin x\cos x = \frac{1}{4},\ \sin 2x = \frac{1}{2}$

recognizing double-angle identity (seen anywhere) ***A1***

eg $\log(\sin 2x),\ 2\sin x\cos x = \sin 2x,\ \sin 2x = \frac{1}{2}$

evaluating $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}\ (30^\circ)$ ***(A1)***

correct working ***A1***

eg $x = \frac{\pi}{12} + 2\pi,\ 2x = \frac{25\pi}{6},\ \frac{29\pi}{6},\ 750^\circ,\ 870^\circ,\ x = \frac{\pi}{12}$ **and** $x = \frac{5\pi}{12}$, one correct final answer

$x = \frac{25\pi}{12},\ \frac{29\pi}{12}$ (do not accept additional values) ***A2 N0***

[7 marks]

Examiners report

[N/A]

The straight line with equation $y = \frac{3}{4}x$ makes an acute angle θ with the x-axis.

a. Write down the value of $\tan\theta$. [1]

b(i) and (ii). Find the value of [6]

(i) $\sin 2\theta$;

(ii) $\cos 2\theta$.

Markscheme

a. $\tan\theta = \frac{3}{4}$ (do not accept $\frac{3}{4}x$) ***A1 N1***

[1 mark]

b(i) and (ii). (i) $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$ ***(A1)(A1)***

correct substitution ***A1***

e.g. $\sin 2\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right)$

$\sin 2\theta = \frac{24}{25}$ ***A1 N3***

(ii) correct substitution ***A1***

e.g. $\cos 2\theta = 1 - 2\left(\frac{3}{5}\right)^2$, $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$\cos 2\theta = \frac{7}{25}$ ***A1 N1***

[6 marks]

Examiners report

a. Many candidates drew a diagram to correctly find $\tan\theta$, although few recognized that a line through the origin can be expressed as $y = x\tan\theta$, with gradient $\tan\theta$, which is explicit in the syllabus.

b(i) and (ii). A surprising number were unable to find the ratios for $\sin\theta$ and $\cos\theta$ from $\tan\theta$. It was not uncommon for candidates to use unreasonable values, such as $\sin\theta = 3$ and $\cos\theta = 4$, or to write nonsense such as $2\sin\frac{3}{5}\cos\frac{4}{5}$.

Let $\sin\theta = \frac{\sqrt{5}}{3}$, where θ is acute.

a. Find $\cos\theta$. [3]

b. Find $\cos 2\theta$. [2]

Markscheme

a. evidence of valid approach ***(M1)***

eg right triangle, $\cos^2\theta = 1 - \sin^2\theta$

correct working ***(A1)***

eg missing side is 2, $\sqrt{1 - \left(\frac{\sqrt{5}}{3}\right)^2}$

$\cos\theta = \frac{2}{3}$ ***A1 N2***

[3 marks]

b. correct substitution into formula for $\cos 2\theta$ ***(A1)***

eg $2 \times \left(\frac{2}{3}\right)^2 - 1,\ 1 - 2\left(\frac{\sqrt{5}}{3}\right)^2,\ \left(\frac{2}{3}\right)^2 - \left(\frac{\sqrt{5}}{3}\right)^2$

$\cos 2\theta = -\frac{1}{9}$ ***A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]

Let $f(x) = \sin^3 x + \cos^3 x \tan x,\ \frac{\pi}{2} < x < \pi$.

a. Show that $f(x) = \sin x$. [2]

b. Let $\sin x = \frac{2}{3}$. Show that $f(2x) = -\frac{4\sqrt{5}}{9}$. [5]

Markscheme

a. changing $\tan x$ into $\frac{\sin x}{\cos x}$ ***A1***

e.g. $\sin^3 x + \cos^3 x \frac{\sin x}{\cos x}$

simplifying ***A1***

e.g $\sin x(\sin^2 x + \cos^2 x)$, $\sin^3 x + \sin x - \sin^3 x$

$f(x) = \sin x$ ***AG N0***

[2 marks]

b. recognizing $f(2x) = \sin 2x$, seen anywhere ***(A1)***

evidence of using double angle identity $\sin(2x) = 2\sin x \cos x$, seen anywhere ***(M1)***

evidence of using Pythagoras with $\sin x = \frac{2}{3}$ ***M1***

e.g. sketch of right triangle, $\sin^2 x + \cos^2 x = 1$

$\cos x = -\frac{\sqrt{5}}{3}$ (accept $\frac{\sqrt{5}}{3}$) ***(A1)***

$f(2x) = 2\left(\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right)$ ***A1***

$f(2x) = -\frac{4\sqrt{5}}{9}$ ***AG*** ***N0***

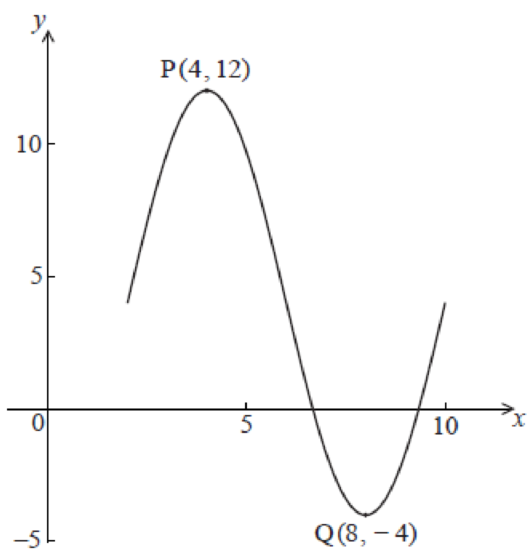
[5 marks]

Examiners report

a. Not surprisingly, this question provided the greatest challenge in section A. In part (a), candidates were able to use the identity $\tan x = \frac{\sin x}{\cos x}$, but many could not proceed any further.

b. Part (b) was generally well done by those candidates who attempted it, the major error arising when the negative sign "magically" appeared in the answer. Many candidates could find the value of cos*x* but failed to observe that cosine is negative in the given domain.

The following diagram shows the graph of $f(x) = a\sin(b(x-c)) + d$, for $2 \le x \le 10$.



There is a maximum point at P(4, 12) and a minimum point at Q(8, −4) .

a(i), (ii) and (iii). Use the graph to write down the value of [3]

(i) a ;

(ii) c ;

(iii) d .

b. Show that $b = \frac{\pi}{4}$. [2]

c. Find $f'(x)$. [3]

d. At a point R, the gradient is -2π . Find the *x*-coordinate of R. [6]

Markscheme

a(i), (ii) and (iii). (i) $a = 8$ ***A1*** ***N1***

(ii) $c = 2$ ***A1*** ***N1***

(iii) $d = 4$ ***A1*** ***N1***

[3 marks]

b. **METHOD 1**

recognizing that period $= 8$ ***(A1)***

correct working ***A1***

e.g. $8 = \frac{2\pi}{b}$, $b = \frac{2\pi}{8}$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

METHOD 2

attempt to substitute ***M1***

e.g. $12 = 8\sin(b(4-2)) + 4$

correct working ***A1***

e.g. $\sin 2b = 1$

$b = \frac{\pi}{4}$ ***AG*** ***N0***

[2 marks]

c. evidence of attempt to differentiate or choosing chain rule ***(M1)***

e.g. $\cos\frac{\pi}{4}(x-2)$, $\frac{\pi}{4} \times 8$

$f'(x) = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$ (accept $2\pi\cos\frac{\pi}{4}(x-2)$) ***A2*** ***N3***

[3 marks]

d. recognizing that gradient is $f'(x)$ ***(M1)***

e.g. $f'(x) = m$

correct equation ***A1***

e.g. $-2\pi = 2\pi\cos\left(\frac{\pi}{4}(x-2)\right)$, $-1 = \cos\left(\frac{\pi}{4}(x-2)\right)$

correct working ***(A1)***

e.g. $\cos^{-1}(-1) = \frac{\pi}{4}(x-2)$

using $\cos^{-1}(-1) = \pi$ (seen anywhere) ***(A1)***

e.g. $\pi = \frac{\pi}{4}(x-2)$

simplifying ***(A1)***

e.g. $4 = (x-2)$

$x = 6$ ***A1*** ***N4***

[6 marks]

Examiners report

a(i), (ii) and (iii). Part (a) of this question proved challenging for most candidates.

b. Although a good number of candidates recognized that the period was 8 in part (b), there were some who did not seem to realize that this period could be found using the given coordinates of the maximum and minimum points.

c. In part (c), not many candidates found the correct derivative using the chain rule.

d. For part (d), a good number of candidates correctly set their expression equal to -2π , but errors in their previous values kept most from correctly solving the equation. Most candidates who had the correct equation were able to gain full marks here.

The expression $6\sin x\cos x$ can be expressed in the form $a\sin bx$.

a. Find the value of a and of b . [3]

b. Hence or otherwise, solve the equation $6\sin x\cos x = \frac{3}{2}$, for $\frac{\pi}{4} \le x \le \frac{\pi}{2}$. [4]

Markscheme

a. recognizing double angle ***M1***

e.g. $3 \times 2\sin x\cos x$, $3\sin 2x$

$a = 3$, $b = 2$ ***A1A1*** ***N3***

[3 marks]

b. substitution $3\sin 2x = \frac{3}{2}$ ***M1***

$\sin 2x = \frac{1}{2}$ ***A1***

finding the angle ***A1***

e.g. $\frac{\pi}{6}$, $2x = \frac{5\pi}{6}$

$x = \frac{5\pi}{12}$ ***A1*** ***N2***

Note: Award ***A0*** if other values are included.

[4 marks]

Examiners report

a. [N/A]

b. [N/A]

---

a. Let $\sin 100^\circ = m$. Find an expression for $\cos 100^\circ$ in terms of m. [3]

b. Let $\sin 100^\circ = m$. Find an expression for $\tan 100^\circ$ in terms of m. [1]

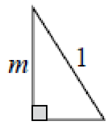
c. Let $\sin 100^\circ = m$. Find an expression for $\sin 200^\circ$ in terms of m. [2]

Markscheme

a. **Note:** All answers must be given in terms of m. If a candidate makes an error that means there is no m in their answer, do not award the final ***A1FT*** mark.

METHOD 1

valid approach involving Pythagoras ***(M1)***

e.g. $\sin^2 x + \cos^2 x = 1$, labelled diagram 

correct working (may be on diagram) ***(A1)***

e.g. $m^2 + (\cos 100)^2 = 1$, $\sqrt{1-m^2}$

$\cos 100 = -\sqrt{1-m^2}$ ***A1 N2***

[3 marks]

METHOD 2

valid approach involving tan identity ***(M1)***

e.g. $\tan = \frac{\sin}{\cos}$

correct working ***(A1)***

e.g. $\cos 100 = \frac{\sin 100}{\tan 100}$

$\cos 100 = \frac{m}{\tan 100}$ ***A1 N2***

[3 marks]

b. **METHOD 1**

$\tan 100 = -\frac{m}{\sqrt{1-m^2}}$ (accept $\frac{m}{-\sqrt{1-m^2}}$) ***A1 N1***

[1 mark]

METHOD 2

$\tan 100 = \frac{m}{\cos 100}$ ***A1 N1***

[1 mark]

c. **METHOD 1**

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta cos\theta$

$\sin 200 = -2m\sqrt{1-m^2}$ (accept $2m\left(-\sqrt{1-m^2}\right)$) ***A1 N2***

Note: If candidates find $\cos 100 = \sqrt{1-m^2}$, award full ***FT*** in parts (b) and (c), even though the values may not have appropriate signs for the angles.

[2 marks]

METHOD 2

valid approach involving double angle formula ***(M1)***

e.g. $\sin 2\theta = 2\sin\theta\cos\theta$, $2m \times \frac{m}{\tan 100}$

$\sin 200 = \frac{2m^2}{\tan 100} (= 2m\cos 100)$ ***A1*** ***N2***

[2 marks]

Examiners report

a. While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2} = 1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100^\circ = m$, then $\sin 200^\circ = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

b. While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2} = 1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100^\circ = m$, then $\sin 200^\circ = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

c. While many candidates correctly approached the problem using Pythagoras in part (a), very few recognized that the cosine of an angle in the second quadrant is negative. Many were able to earn follow-through marks in subsequent parts of the question. A common algebraic error in part (a) was for candidates to write $\sqrt{1-m^2} = 1-m$. In part (c), many candidates failed to use the double-angle identity. Many incorrectly assumed that because $\sin 100^\circ = m$, then $\sin 200^\circ = 2m$. In addition, some candidates did not seem to understand what writing an expression "in terms of *m*" meant.

Let $\int_\pi^a \cos 2x \mathrm{d}x = \frac{1}{2}$, where $\pi < a < 2\pi$. Find the value of a.

Markscheme

correct integration (ignore absence of limits and "$+C$") ***(A1)***

eg $\frac{\sin(2x)}{2}$, $\int_\pi^a \cos 2x = \left[\frac{1}{2}\sin(2x)\right]_\pi^a$

substituting limits into **their** integrated function and subtracting (in any order) ***(M1)***

eg $\frac{1}{2}\sin(2a) - \frac{1}{2}\sin(2\pi),\ \sin(2\pi) - \sin(2a)$

$\sin(2\pi) = 0$ ***(A1)***

setting **their** result from an integrated function equal to $\frac{1}{2}$ ***M1***

eg $\frac{1}{2}\sin 2a = \frac{1}{2},\ \sin(2a) = 1$

recognizing $\sin^{-1} 1 = \frac{\pi}{2}$ ***(A1)***

eg $2a = \frac{\pi}{2},\ a = \frac{\pi}{4}$

correct value ***(A1)***

eg $\frac{\pi}{2} + 2\pi,\ 2a = \frac{5\pi}{2},\ a = \frac{\pi}{4} + \pi$

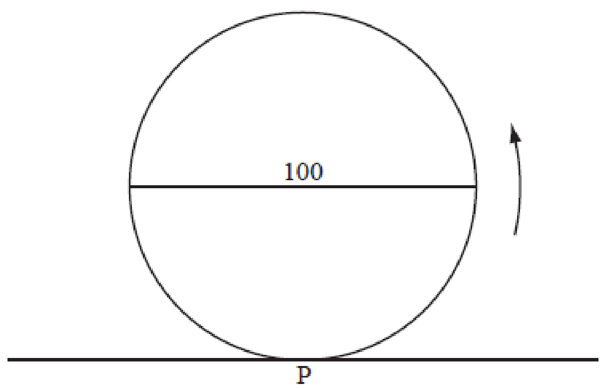
$a = \frac{5\pi}{4}$ ***A1*** ***N3***

[7 marks]

Examiners report

[N/A]

The following diagram represents a large Ferris wheel, with a diameter of 100 metres.



Let P be a point on the wheel. The wheel starts with P at the lowest point, at ground level. The wheel rotates at a constant rate, in an anticlockwise (counter-clockwise) direction. One revolution takes 20 minutes.

Let $h(t)$ metres be the height of P above ground level after *t* minutes. Some values of $h(t)$ are given in the table below.

t	$h(t)$
0	0.0
1	2.4
2	9.5
3	20.6
4	34.5
5	50.0

a(i) and (ii). Write down the height of P above ground level after [2]

(i) 10 minutes;

(ii) 15 minutes.

b(i) and (ii). (i) Show that $h(8) = 90.5$. [4]

(ii) Find $h(21)$.

c. **Sketch** the graph of h , for $0 \leq t \leq 40$. [3]

d. Given that h can be expressed in the form $h(t) = a\cos bt + c$, find a , b and c . [5]

Markscheme

a(i) and (ii). (i) 100 (metres) ***A1*** ***N1***

(ii) 50 (metres) ***A1*** ***N1***

[2 marks]

b(i) and (ii). (i) identifying symmetry with $h(2) = 9.5$ ***(M1)***

subtraction ***A1***

e.g. $100 - h(2)$, $100 - 9.5$

$h(8) = 90.5$ ***AG*** ***N0***

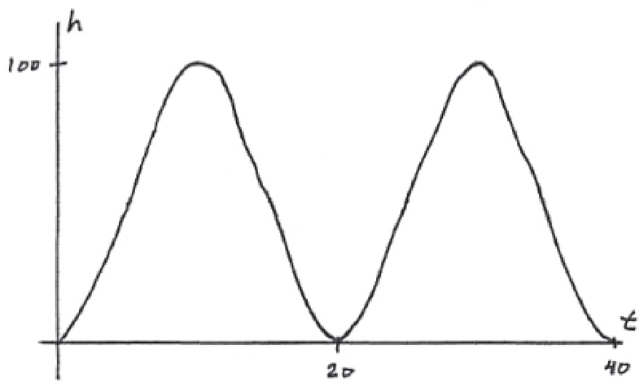
(ii) recognizing period ***(M1)***

e.g. $h(21) = h(1)$

$h(21) = 2.4$ ***A1*** ***N2***

[4 marks]

c.



A1A1A1 ***N3***

Note: Award ***A1*** for end points (0, 0) and (40, 0) , ***A1*** for range $0 \le h \le 100$, ***A1*** for approximately correct sinusoidal shape, with two cycles.

[3 marks]

d. evidence of a quotient involving 20, 2π or 360° to find b ***(M1)***

e.g. $\frac{2\pi}{b} = 20$, $b = \frac{360}{20}$

$b = \frac{2\pi}{20}\left(= \frac{\pi}{10}\right)$ (accept $b = 18$ if working in degrees) ***A1*** ***N2***

$a = -50$, $c = 50$ ***A2A1*** ***N3***

[5 marks]

Examiners report

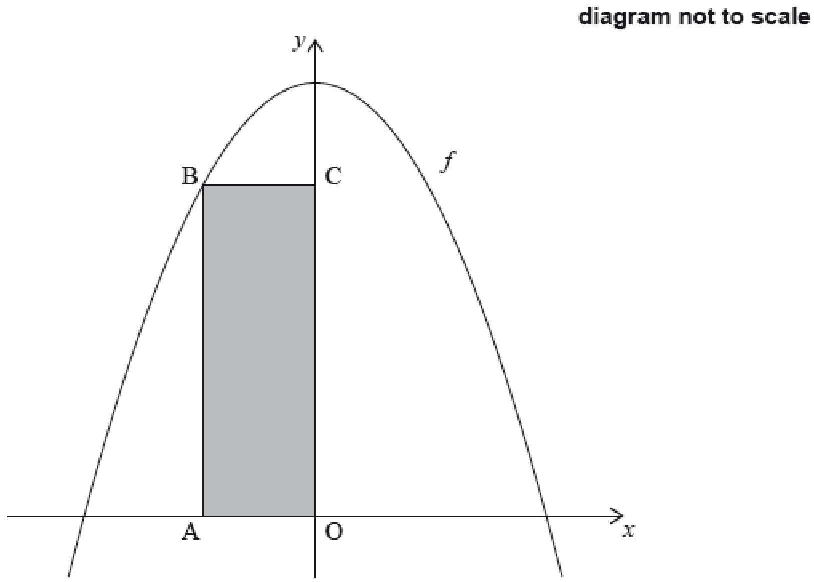
a(i) and (ii). Nearly all candidates answered part (a) correctly, finding the height of the wheel at $\frac{1}{2}$ and $\frac{3}{4}$ of a revolution.

b(i) and (ii). While many candidates were successful in part (b), there were many who tried to use right-angled triangles or find a function for height, rather than recognizing the symmetry of the wheel in its different positions and using the values given in the table.

c. In part (c), most candidates were able to sketch a somewhat accurate representation of the height of the wheel over two full cycles. However, it seems that many candidates are not familiar with the shape of a sinusoidal wave, as many of the candidates' graphs were constructed of line segments, rather than a curve.

d. For part (d), candidates were less successful in finding the parameters of the cosine function. Even candidates who drew accurate sketches were not always able to relate their sketch to the function. These candidates understood the context of the problem, that the position on the wheel goes up and down, but they did not relate this to a trigonometric function. Only a small number of candidates recognized that the value of ***a*** would be negative. Candidates should be aware that while working in degrees may be acceptable, the expectation is that radians will be used in these types of questions.

Let $f(x) = 15 - x^2$, for $x \in \mathbb{R}$. The following diagram shows part of the graph of f and the rectangle OABC, where A is on the negative x-axis, B is on the graph of f, and C is on the y-axis.



Find the x-coordinate of A that gives the maximum area of OABC.

Markscheme

attempt to find the area of OABC ***(M1)***

eg $\mathrm{OA} \times \mathrm{OC}, x \times f(x),\ f(x) \times (-x)$

correct expression for area in one variable ***(A1)***

eg $\text{area} = x(15 - x^2),\ 15x - x^3,\ x^3 - 15x$

valid approach to find maximum **area** (seen anywhere) ***(M1)***

eg $A'(x) = 0$

correct derivative ***A1***

eg $15 - 3x^2,\ (15 - x^2) + x(-2x) = 0,\ -15 + 3x^2$

correct working ***(A1)***

eg $15 = 3x^2,\ x^2 = 5,\ x = \sqrt{5}$

$x = -\sqrt{5}\ \left(\text{accept A}\left(-\sqrt{5},\ 0\right)\right)$ ***A2*** ***N3***

[7 marks]

Examiners report

[N/A]

Let $f(x) = \sin\left(x + \frac{\pi}{4}\right) + k$. The graph of f passes through the point $\left(\frac{\pi}{4},\ 6\right)$.

a. Find the value of k. [3]

b. Find the minimum value of $f(x)$. [2]

c. Let $g(x) = \sin x$. The graph of g is translated to the graph of f by the vector $\begin{pmatrix} p \\ q \end{pmatrix}$. [2]

Write down the value of p and of q.

Markscheme

a. **METHOD 1**

attempt to substitute both coordinates (in any order) into f ***(M1)***

eg $f\left(\frac{\pi}{4}\right) = 6,\ \frac{\pi}{4} = \sin\left(6 + \frac{\pi}{4}\right) + k$

correct working ***(A1)***

eg $\sin\frac{\pi}{2} = 1,\ 1 + k = 6$

$k = 5$ ***A1 N2***

[3 marks]

METHOD 2

recognizing shift of $\frac{\pi}{4}$ left means maximum at 6 ***R1)***

recognizing k is difference of maximum and amplitude ***(A1)***

eg $6 - 1$

$k = 5$ ***A1 N2***

[3 marks]

b. evidence of appropriate approach ***(M1)***

eg minimum value of $\sin x$ is $-1,\ -1 + k,\ f'(x) = 0,\ \left(\frac{5\pi}{4},\ 4\right)$

minimum value is 4 ***A1 N2***

[2 marks]

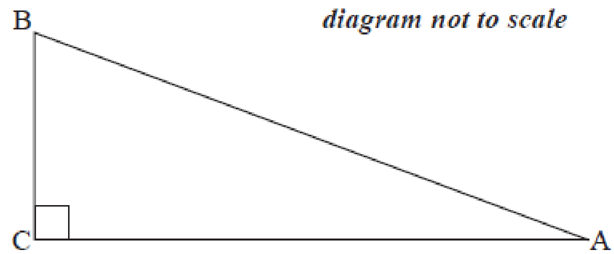
c. $p = -\frac{\pi}{4},\ q = 5\ \left(\text{accept } \begin{pmatrix} -\frac{\pi}{4} \\ 5 \end{pmatrix}\right)$ ***A1A1 N2***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The following diagram shows a right-angled triangle, ABC, where $\sin \mathrm{A} = \frac{5}{13}$.



a. Show that $\cos A = \frac{12}{13}$. [2]

b. Find $\cos 2A$. [3]

Markscheme

a. **METHOD 1**

approach involving Pythagoras' theorem ***(M1)***

eg $5^2 + x^2 = 13^2$, labelling correct sides on triangle

finding third side is 12 (may be seen on diagram) ***A1***

$\cos A = \frac{12}{13}$ ***AG*** ***N0***

METHOD 2

approach involving $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

eg $\left(\frac{5}{13}\right)^2 + \cos^2\theta = 1,\ x^2 + \frac{25}{169} = 1$

correct working ***A1***

eg $\cos^2\theta = \frac{144}{169}$

$\cos A = \frac{12}{13}$ ***AG*** ***N0***

[2 marks]

b. correct substitution into $\cos 2\theta$ ***(A1)***

eg $1 - 2\left(\frac{5}{13}\right)^2,\ 2\left(\frac{12}{13}\right)^2 - 1,\ \left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2$

correct working ***(A1)***

eg $1 - \frac{50}{169},\ \frac{288}{169} - 1,\ \frac{144}{169} - \frac{25}{169}$

$\cos 2A = \frac{119}{169}$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]

b. [N/A]

Let $f(x) = \cos 2x$ and $g(x) = 2x^2 - 1$.

a. Find $f\left(\frac{\pi}{2}\right)$. [2]

b. Find $(g \circ f)\left(\frac{\pi}{2}\right)$. [2]

c. Given that $(g \circ f)(x)$ can be written as $\cos(kx)$, find the value of k, $k \in \mathbb{Z}$. [3]

Markscheme

a. $f\left(\frac{\pi}{2}\right) = \cos \pi$ ***(A1)***

$= -1$ ***A1 N2***

[2 marks]

b. $(g \circ f)\left(\frac{\pi}{2}\right) = g(-1)\ (= 2(-1)^2 - 1)$ ***(A1)***

$= 1$ ***A1 N2***

[2 marks]

c. $(g \circ f)(x) = 2(\cos(2x))^2 - 1\ (= 2\cos^2(2x) - 1)$ ***A1***

evidence of $2\cos^2\theta - 1 = \cos 2\theta$ (seen anywhere) ***(M1)***

$(g \circ f)(x) = \cos 4x$

$k = 4$ ***A1 N2***

[3 marks]

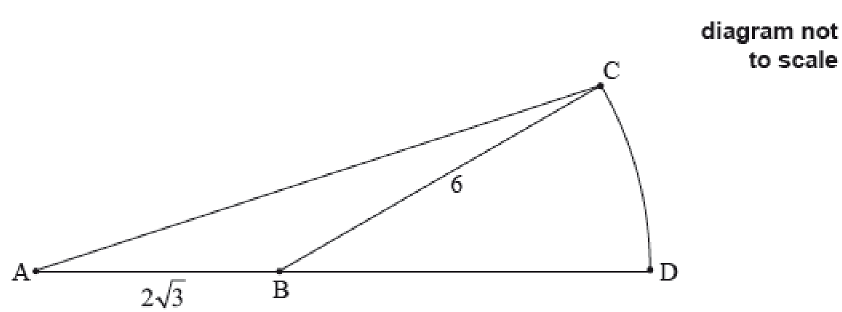
Examiners report

a. In part (a), a number of candidates were not able to evaluate $\cos \pi$, either leaving it or evaluating it incorrectly.

b. Almost all candidates evaluated the composite function in part (b) in the given order, many earning follow-through marks for incorrect answers from part (a). On both parts (a) and (b), there were candidates who correctly used double-angle formulas to come up with correct answers; while this is a valid method, it required unnecessary additional work.

c. Candidates were not as successful in part (c). Many tried to use double-angle formulas, but either used the formula incorrectly or used it to write the expression in terms of $\cos x$ and went no further. There were a number of cases in which the candidates "accidentally" came up with the correct answer based on errors or lucky guesses and did not earn credit for their final answer. Only a few candidates recognized the correct method of solution.

The following diagram shows a triangle ABC and a sector BDC of a circle with centre B and radius 6 cm. The points A , B and D are on the same line.



$\mathrm{AB} = 2\sqrt{3}$ cm, $\mathrm{BC} = 6$ cm, area of triangle $\mathrm{ABC} = 3\sqrt{3}\ \mathrm{cm}^2$, $\mathrm{A\hat{B}C}$ is obtuse.

a. Find $\mathrm{A\hat{B}C}$. [5]

b. Find the exact area of the sector BDC. [3]

Markscheme

a. **METHOD 1**

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B,\ 6\sqrt{3}\sin B,\ \frac{1}{2}(6)\left(2\sqrt{3}\right)\sin B = 3\sqrt{3}$

correct working ***(A1)***

eg $6\sqrt{3}\sin B = 3\sqrt{3},\ \sin B = \frac{3\sqrt{3}}{\frac{1}{2}(6)2\sqrt{3}}$

$\sin B = \frac{1}{2}$ ***(A1)***

$\frac{\pi}{6}(30^\circ)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150^\circ)$ ***A1*** ***N3***

METHOD 2

(using height of triangle ABC by drawing perpendicular segment from C to AD)

correct substitution into formula for area of triangle ***(A1)***

eg $\frac{1}{2}\left(2\sqrt{3}\right)(h) = 3\sqrt{3},\ h\sqrt{3}$

correct working ***(A1)***

eg $h\sqrt{3} = 3\sqrt{3}$

height of triangle is 3 ***A1***

$\mathrm{C\hat{B}D} = \frac{\pi}{6}(30^\circ)$ ***(A1)***

$\mathrm{A\hat{B}C} = \frac{5\pi}{6}(150^\circ)$ ***A1*** ***N3***

[5 marks]

b. recognizing supplementary angle ***(M1)***

eg $\mathrm{C\hat{B}D} = \frac{\pi}{6},\ \text{sector} = \frac{1}{2}(180 - \mathrm{A\hat{B}C})(6^2)$

correct substitution into formula for area of sector ***(A1)***

eg $\frac{1}{2}\times\frac{\pi}{6}\times 6^2,\ \pi(6^2)\left(\frac{30}{360}\right)$

$\text{area} = 3\pi\ (\text{cm}^2)$ ***A1*** ***N2***

[3 marks]

Examiners report

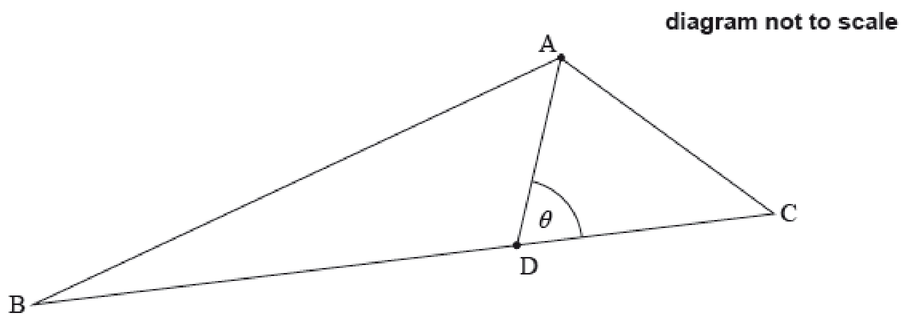
a. In part (a) of this question, the large majority of candidates substituted correctly into the area formula for the triangle, though algebraic errors kept some of them from simplifying the equation to $\sin\mathrm{A\hat{B}C} = \frac{1}{2}$. Unfortunately, a number of candidates who got to this point often did not know the correct angles that correspond with this sine value.

b. In part (b), many candidates realized that $\mathrm{C\hat{B}D}$ was the supplement of $\mathrm{A\hat{B}C}$. However, at this point many candidates substituted 30°, or their follow-through angle in degrees, into the formula for the area of a sector found in the formula booklet, not understanding that this formula only works for angles in radians.

Let $\overrightarrow{\mathrm{OA}} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$ and $\overrightarrow{\mathrm{OB}} = \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix}$.

The point C is such that $\overrightarrow{\mathrm{AC}} = \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix}$.

The following diagram shows triangle ABC. Let D be a point on [BC], with acute angle $\mathrm{ADC} = \theta$.



a. (i) Find $\overrightarrow{\mathrm{AB}}$. [4]

(ii) Find $\left|\overrightarrow{\mathrm{AB}}\right|$.

b. Show that the coordinates of C are $(-2,\ 1,\ 3)$. [1]

c. Write down an expression in terms of θ for [2]

(i) angle ADB;

(ii) area of triangle ABD.

d. Given that $\frac{\text{area }\Delta\mathrm{ABD}}{\text{area }\Delta\mathrm{ACD}} = 3$, show that $\frac{\mathrm{BD}}{\mathrm{BC}} = \frac{3}{4}$. [5]

e. Hence or otherwise, find the coordinates of point D. [4]

Markscheme

a. (i) valid approach to find $\overrightarrow{\mathrm{AB}}$

eg $\overrightarrow{\mathrm{OB}} - \overrightarrow{\mathrm{OA}},\ \begin{pmatrix} 4-(-1) \\ 1-0 \\ 3-4 \end{pmatrix}$

$\overrightarrow{\mathrm{AB}} = \begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ ***A1 N2***

(ii) valid approach to find $\left|\overrightarrow{\mathrm{AB}}\right|$ ***(M1)***

eg $\sqrt{(5)^2+(1)^2+(-1)^2}$

$\left|\overrightarrow{\mathrm{AB}}\right|=\sqrt{27}$ ***A1 N2***

[4 marks]

b. correct approach ***A1***

eg $\overrightarrow{\mathrm{OC}}=\begin{pmatrix}-1\\1\\-1\end{pmatrix}+\begin{pmatrix}-1\\0\\4\end{pmatrix}$

C has coordinates $(-2,\ 1,\ 3)$ ***AG N0***

[1 mark]

c. (i) $\mathrm{A\hat{D}B}=\pi-\theta, \hat{\mathrm{D}}=180-\theta$ ***A1 N1***

(ii) any correct expression for the area involving θ ***A1 N1***

eg $\text{area}=\frac{1}{2}\times\mathrm{AD}\times\mathrm{BD}\times\sin(180-\theta),\ \frac{1}{2}ab\sin\theta,\ \frac{1}{2}\left|\overrightarrow{\mathrm{DA}}\right|\left|\overrightarrow{\mathrm{DB}}\right|\sin(\pi-\theta)$

[2 marks]

d. **METHOD 1** (using sine formula for area)

correct expression for the area of triangle ACD (seen anywhere) ***(A1)***

eg $\frac{1}{2}\mathrm{AD}\times\mathrm{DC}\times\sin\theta$

correct equation involving areas ***A1***

eg $\frac{\frac{1}{2}\mathrm{AD}\times\mathrm{BD}\times\sin(\pi-\theta)}{\frac{1}{2}\mathrm{AD}\times\mathrm{DC}\times\sin\theta}=3$

recognizing that $\sin(\pi-\theta)=\sin\theta$ (seen anywhere) ***(A1)***

$\frac{\mathrm{BD}}{\mathrm{DC}}=3$ (seen anywhere) ***(A1)***

correct approach using ratio ***A1***

eg $3\overrightarrow{\mathrm{DC}}+\overrightarrow{\mathrm{DC}}=\overrightarrow{\mathrm{BC}},\ \overrightarrow{\mathrm{BC}}=4\overrightarrow{\mathrm{DC}}$

correct ratio $\frac{\mathrm{BD}}{\mathrm{BC}}=\frac{3}{4}$ ***AG N0***

METHOD 2 (Geometric approach)

recognising $\Delta\mathrm{ABD}$ and $\Delta\mathrm{ACD}$ have same height ***(A1)***

eg use of h for both triangles, $\frac{\frac{1}{2}\mathrm{BD}\times h}{\frac{1}{2}\mathrm{CD}\times h}=3$

correct approach ***A2***

eg $\mathrm{BD}=3x$ and $\mathrm{DC}=x,\ \frac{\mathrm{BD}}{\mathrm{DC}}=3$

correct working ***A2***

eg $\mathrm{BC}=4x,\ \mathrm{BD}+\mathrm{DC}=4\mathrm{DC},\ \frac{\mathrm{BD}}{\mathrm{BC}}=\frac{3x}{4x},\ \frac{\mathrm{BD}}{\mathrm{BC}}=\frac{3\mathrm{DC}}{4\mathrm{DC}}$

$\frac{\mathrm{BD}}{\mathrm{BC}}=\frac{3}{4}$ ***AG N0***

[5 marks]

e. correct working (seen anywhere) ***(A1)*** *eg* $\overrightarrow{\mathrm{BD}}=\frac{3}{4}\overrightarrow{\mathrm{BC}},\ \overrightarrow{\mathrm{OD}}=\overrightarrow{\mathrm{OB}}+\frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix},\ \overrightarrow{\mathrm{CD}}=\frac{1}{4}\overrightarrow{\mathrm{CB}}$

valid approach (seen anywhere) ***(M1)***

eg $\overrightarrow{\mathrm{OD}}=\overrightarrow{\mathrm{OB}}+\overrightarrow{\mathrm{BD}},\ \overrightarrow{\mathrm{BC}}=\begin{pmatrix}-6\\0\\0\end{pmatrix}$

correct working to find x-coordinate ***(A1)***

eg $\begin{pmatrix}4\\1\\3\end{pmatrix}+\frac{3}{4}\begin{pmatrix}-6\\0\\0\end{pmatrix},\ x=4+\frac{3}{4}(-6),\ -2+\frac{1}{4}(6)$

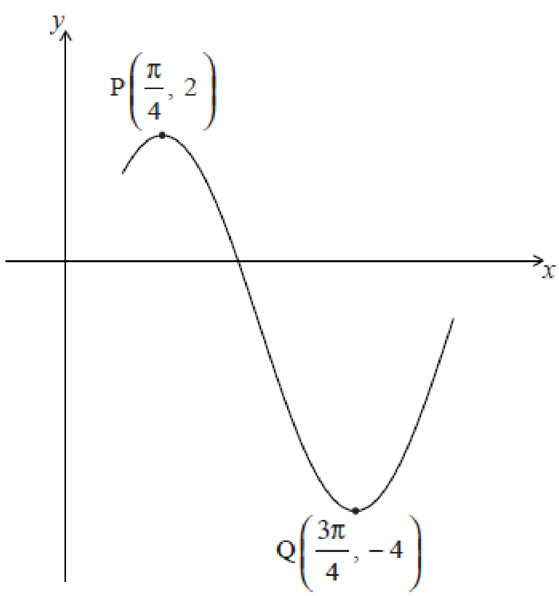
D is $\left(-\frac{1}{2},\ 1,\ 3\right)$ ***A1 N3***

[4 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]
e. [N/A]

The diagram below shows part of the graph of $f(x)=a\cos(b(x-c))-1$, where $a>0$.



The point P $\left(\frac{\pi}{4},2\right)$ is a maximum point and the point Q $\left(\frac{3\pi}{4},-4\right)$ is a minimum point.

a. Find the value of a . [2]

b(i) and (ii). (i) Show that the period of f is π . [4]

(ii) Hence, find the value of b .

c. Given that $0 < c < \pi$, write down the value of c . [1]

Markscheme

a. evidence of valid approach ***(M1)***

e.g. $\frac{\max y \text{ value} - \min y \text{ value}}{2}$, distance from $y = -1$

$a = 3$ ***A1 N2***

[2 marks]

b(i) and (ii). (i) evidence of valid approach ***(M1)***

e.g. finding difference in x-coordinates, $\frac{\pi}{2}$

evidence of doubling ***A1***

e.g. $2 \times \left(\frac{\pi}{2}\right)$

period $= \pi$ ***AG N0***

(ii) evidence of valid approach ***(M1)***

e.g. $b = \frac{2\pi}{\pi}$

$b = 2$ ***A1 N2***

[4 marks]

c. $c = \frac{\pi}{4}$ ***A1 N1***

[1 mark]

Examiners report

a. A pleasing number of candidates correctly found the values of a, b, and c for this sinusoidal graph.

b(i) and (ii). A pleasing number of candidates correctly found the values of a, b, and c for this sinusoidal graph. Some candidates had trouble showing that the period was π , either incorrectly adding the given $\pi/4$ and $3\pi/4$ or using the value of b that they found first for part (b)(ii).

c. A pleasing number of candidates correctly found the values of a, b, and c for this sinusoidal graph. Some candidates had trouble showing that the period was π , either incorrectly adding the given $\pi/4$ and $\pi/3$ or using the value of b that they found first for part (b)(ii).

Let $p = \sin 40^\circ$ and $q = \cos 110^\circ$. Give your answers to the following in terms of p and/or q .

a(i) and (ii). Write down an expression for [2]

(i) $\sin 140^\circ$;

(ii) $\cos 70^\circ$.

b. Find an expression for $\cos 140^\circ$. [3]

c. Find an expression for $\tan 140^\circ$. [1]

Markscheme

a(i) and (ii) (i) $\sin 140^\circ = p$ ***A1 N1***

(ii) $\cos 70^\circ = -q$ ***A1 N1***

[2 marks]

b. **METHOD 1**

evidence of using $\sin^2\theta + \cos^2\theta = 1$ ***(M1)***

e.g. diagram, $\sqrt{1-p^2}$ (seen anywhere)

$\cos 140^\circ = \pm\sqrt{1-p^2}$ ***(A1)***

$\cos 140^\circ = -\sqrt{1-p^2}$ ***A1 N2***

METHOD 2

evidence of using $\cos 2\theta = 2\cos^2\theta - 1$ ***(M1)***

$\cos 140^\circ = 2\cos^2 70 - 1$ ***(A1)***

$\cos 140^\circ = 2(-q)^2 - 1\ (= 2q^2 - 1)$ ***A1 N2***

[3 marks]

c. **METHOD 1**

$\tan 140^\circ = \frac{\sin 140^\circ}{\cos 140^\circ} = -\frac{p}{\sqrt{1-p^2}}$ ***A1 N1***

METHOD 2

$\tan 140^\circ = \frac{p}{2q^2-1}$ ***A1 N1***

[1 mark]

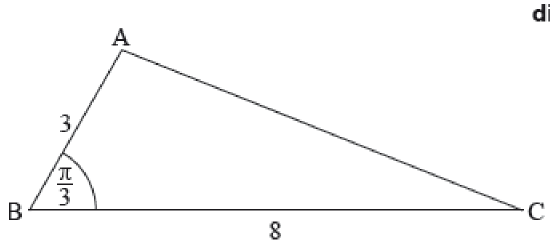
Examiners report

a(i) and (ii) This was one of the most difficult problems for the candidates. Even the strongest candidates had a hard time with this one and only a few received any marks at all.

b. Many did not appear to know the relationships between trigonometric functions of supplementary angles and that the use of $\sin^2 x + \cos^2 x = 1$ results in a $\pm$ value. The application of a double angle formula also seemed weak.

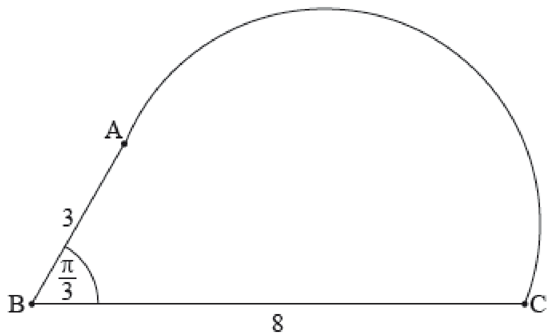
c. This was one of the most difficult problems for the candidates. Even the strongest candidates had a hard time with this one and only a few received any marks at all. Many did not appear to know the relationships between trigonometric functions of supplementary angles and that the use of $\sin^2 x + \cos^2 x = 1$ results in a $\pm$ value. The application of a double angle formula also seemed weak.

The following diagram shows triangle ABC, with $\mathrm{AB} = 3\text{ cm}$, $\mathrm{BC} = 8\text{ cm}$, and $\mathrm{A\hat{B}C} = \frac{\pi}{3}$.



a. Show that $\mathrm{AC} = 7\ \mathrm{cm}$. [4]

b. The shape in the following diagram is formed by adding a semicircle with diameter [AC] to the triangle. [3]



Find the exact perimeter of this shape.

Markscheme

a. evidence of choosing the cosine rule ***(M1)***

eg $c^2 = a^2 + b^2 - ab\cos C$

correct substitution into RHS of cosine rule ***(A1)***

eg $3^2 + 8^2 - 2 \times 3 \times 8 \times \cos\frac{\pi}{3}$

evidence of correct value for $\cos\frac{\pi}{3}$ (may be seen anywhere, including in cosine rule) ***A1***

eg $\cos\frac{\pi}{3} = \frac{1}{2},\ \mathrm{AC}^2 = 9 + 64 - \left(48 \times \frac{1}{2}\right),\ 9 + 64 - 24$

correct working clearly leading to answer ***A1***

eg $\mathrm{AC}^2 = 49,\ b = \sqrt{49}$

$\mathrm{AC} = 7\ (\mathrm{cm})$ ***AG*** ***N0***

Note: Award no marks if the only working seen is $\mathrm{AC}^2 = 49$ or $\mathrm{AC} = \sqrt{49}$ (or similar).

[4 marks]

b. correct substitution for semicircle ***(A1)***

eg $\text{semicircle} = \frac{1}{2}(2\pi \times 3.5),\ \frac{1}{2} \times \pi \times 7,\ 3.5\pi$

valid approach (seen anywhere) ***(M1)***

eg $\text{perimeter} = \mathrm{AB} + \mathrm{BC} + \text{semicircle},\ 3 + 8 + \left(\frac{1}{2} \times 2 \times \pi \times \frac{7}{2}\right),\ 8 + 3 + 3.5\pi$

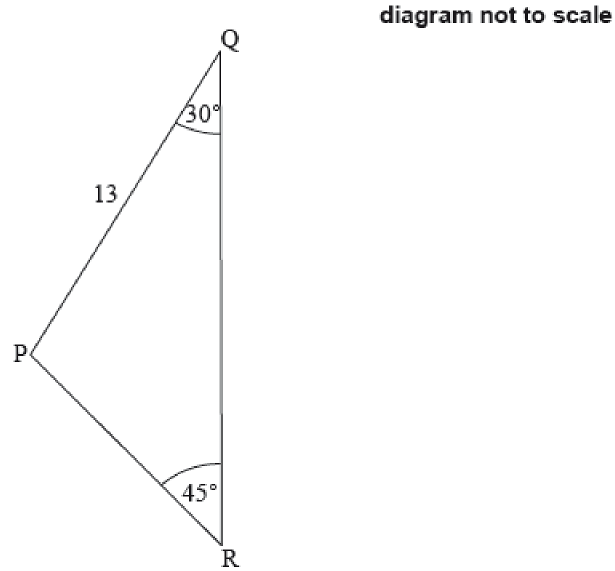
$11 + \frac{7}{2}\pi\ (= 3.5\pi + 11)\ (\mathrm{cm})$ ***A1*** ***N2***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

The following diagram shows triangle PQR.



$\hat{PQR} = 30°$, $\hat{QRP} = 45°$ and $PQ = 13\text{ cm}$.

Find PR.

Markscheme

METHOD 1

evidence of choosing the sine rule ***(M1)***

eg $\frac{a}{\sin A} = \frac{b}{\sin B}$

correct substitution ***A1***

eg $\frac{x}{\sin 30} = \frac{13}{\sin 45}$, $\frac{13 \sin 30}{\sin 45}$

$\sin 30 = \frac{1}{2}$, $\sin 45 = \frac{1}{\sqrt{2}}$ ***(A1)(A1)***

correct working ***A1***

eg $\frac{1}{2} \times \frac{13}{\frac{1}{\sqrt{2}}}$, $\frac{1}{2} \times 13 \times \frac{2}{\sqrt{2}}$, $13 \times \frac{1}{2} \times \sqrt{2}$

correct answer ***A1 N3***

eg $\text{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

METHOD 2 (using height of ΔPQR)

valid approach to find height of ΔPQR ***(M1)***

eg $\sin 30 = \frac{x}{13}$, $\cos 60 = \frac{x}{13}$

$\sin 30 = \frac{1}{2}$ or $\cos 60 = \frac{1}{2}$ ***(A1)***

$\text{height} = 6.5$ ***A1***

correct working ***A1***

eg $\sin 45 = \frac{6.5}{\mathrm{PR}}$, $\sqrt{6.5^2 + 6.5^2}$

correct working ***(A1)***

eg $\sin 45 = \frac{1}{\sqrt{2}}$, $\cos 45 = \frac{1}{\sqrt{2}}$, $\sqrt{\frac{169\times 2}{4}}$

correct answer ***A1*** ***N3***

eg $\mathrm{PR} = \frac{13\sqrt{2}}{2}$, $\frac{13}{\sqrt{2}}$ (cm)

[6 marks]

Examiners report

[N/A]

Let $f(x) = \mathrm{e}^{-3x}$ and $g(x) = \sin\left(x - \frac{\pi}{3}\right)$.

a. Write down [2]

(i) $f'(x)$;

(ii) $g'(x)$.

b. Let $h(x) = \mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right)$. Find the exact value of $h'\left(\frac{\pi}{3}\right)$. [4]

Markscheme

a. (i) $-3\mathrm{e}^{-3x}$ ***A1*** ***N1***

(ii) $\cos\left(x - \frac{\pi}{3}\right)$ ***A1*** ***N1***

[4 marks]

b. evidence of choosing product rule ***(M1)***

e.g. $uv' + vu'$

correct expression ***A1***

e.g. $-3\mathrm{e}^{-3x}\sin\left(x - \frac{\pi}{3}\right) + \mathrm{e}^{-3x}\cos\left(x - \frac{\pi}{3}\right)$

complete correct substitution of $x = \frac{\pi}{3}$ ***(A1)***

e.g. $-3\mathrm{e}^{-3\frac{\pi}{3}}\sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) + \mathrm{e}^{-3\frac{\pi}{3}}\cos\left(\frac{\pi}{3} - \frac{\pi}{3}\right)$□□□□□□□□□

$h'\left(\frac{\pi}{3}\right) = \mathrm{e}^{-\pi}$ ***A1*** ***N3***

[4 marks]

Examiners report

a. A good number of candidates found the correct derivative expressions in (a). Many applied the product rule, although with mixed success.

b. Often the substitution of $\frac{\pi}{3}$ was incomplete or not done at all.

a. Given that $\cos A = \frac{1}{3}$ and $0 \le A \le \frac{\pi}{2}$, find $\cos 2A$. [3]

b. Given that $\sin B = \frac{2}{3}$ and $\frac{\pi}{2} \le B \le \pi$, find $\cos B$. [3]

Markscheme

a. evidence of choosing the formula $\cos 2A = 2\cos^2 A - 1$ ***(M1)***

Note: If they choose another correct formula, do not award the ***M1*** unless there is evidence of finding $\sin^2 A = 1 - \frac{1}{9}$

correct substitution ***A1***

e.g. $\cos 2A = \left(\frac{1}{3}\right)^2 - \frac{8}{9}$, $\cos 2A = 2 \times \left(\frac{1}{3}\right)^2 - 1$

$\cos 2A = -\frac{7}{9}$ ***A1*** ***N2***

[3 marks]

b. **METHOD 1**

evidence of using $\sin^2 B + \cos^2 B = 1$ ***(M1)***

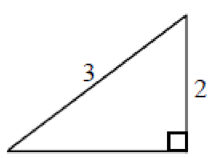
e.g. $\left(\frac{2}{3}\right)^2 + \cos^2 B = 1$, $\sqrt{\frac{5}{9}}$ (seen anywhere),

$\cos B = \pm\sqrt{\frac{5}{9}} \left(= \pm\frac{\sqrt{5}}{3}\right)$ ***(A1)***

$\cos B = -\sqrt{\frac{5}{9}} \left(= -\frac{\sqrt{5}}{3}\right)$ ***A1*** ***N2***

METHOD 2

diagram ***M1***



for finding third side equals $\sqrt{5}$ ***(A1)***

$\cos B = -\frac{\sqrt{5}}{3}$ ***A1*** ***N2***

[3 marks]

Examiners report

a. This question was very poorly done, and knowledge of basic trigonometric identities and values of trigonometric functions of obtuse angles seemed distinctly lacking. Candidates who recognized the need of an identity for finding $\cos 2A$ given $\cos A$ seldom chose the most appropriate of the three and even when they did often used it incorrectly with expressions such as $2\cos^2 \frac{1}{9} - 1$.

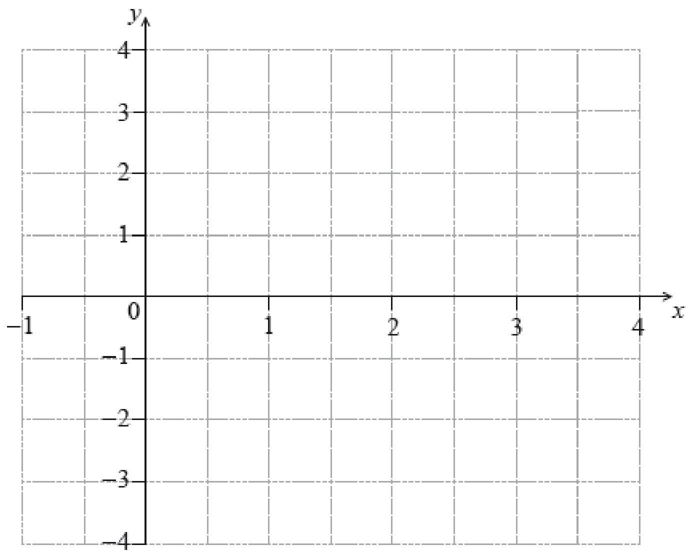
b. This question was very poorly done, and knowledge of basic trigonometric identities and values of trigonometric functions of obtuse angles seemed distinctly lacking. Candidates who recognized the need of an identity for finding $\cos 2A$ given $\cos A$ seldom chose the most appropriate of the three and even when they did often used it incorrectly with expressions such as $2\cos^2 \frac{1}{9} - 1$.

Let $f(x) = 3\sin(\pi x)$.

a. Write down the amplitude of f. [1]

b. Find the period of f. [2]

c. On the following grid, sketch the graph of $y = f(x)$, for $0 \leq x \leq 3$. [4]



Markscheme

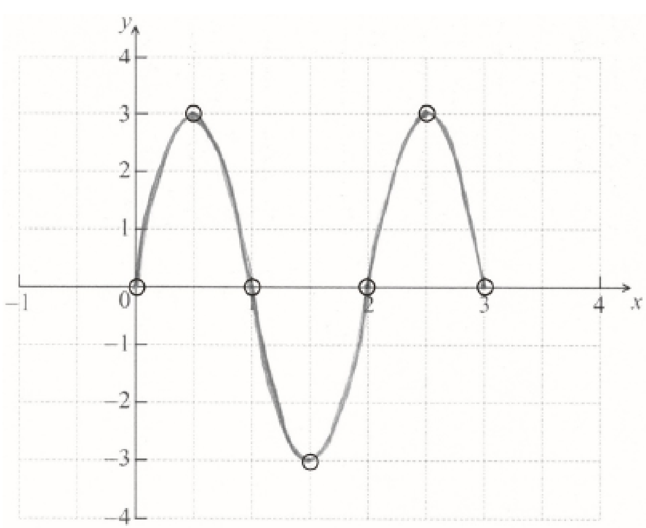
a. amplitude is 3 ***A1*** ***N1***

b. valid approach ***(M1)***

eg $\text{period} = \frac{2\pi}{\pi},\ \frac{360}{\pi}$

period is 2 ***A1*** ***N2***

c.



A1

A1A1A1 ***N4***

Note: Award ***A1*** for sine curve starting at (0, 0) and correct period.

Only if this ***A1*** is awarded, award the following for points in circles:

A1 for correct x-intercepts;

A1 for correct max and min points;

A1 for correct domain.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The first two terms of an infinite geometric sequence are $u_1 = 18$ and $u_2 = 12\sin^2\theta$, where $0 < \theta < 2\pi$, and $\theta \ne \pi$.

a.i. Find an expression for r in terms of θ. [2]

a.ii. Find the possible values of r. [3]

b. Show that the sum of the infinite sequence is $\frac{54}{2+\cos(2\theta)}$. [4]

c. Find the values of θ which give the greatest value of the sum. [6]

Markscheme

a.i. valid approach ***(M1)***

eg $\frac{u_2}{u_1}, \frac{u_1}{u_2}$

$r = \frac{12\sin^2\theta}{18}\left(= \frac{2\sin^2\theta}{3}\right)$ ***A1 N2***

[2 marks]

a.ii. recognizing that $\sin\theta$ is bounded ***(M1)***

eg $0 \le \sin^2\theta \le 1$, $-1 \le \sin\theta \le 1$, $-1 < \sin\theta < 1$

$0 < r \le \frac{2}{3}$ ***A2 N3***

Note: If working shown, award ***M1A1*** for correct values with incorrect inequality sign(s).
If no working shown, award ***N1*** for correct values with incorrect inequality sign(s).

[3 marks]

b. correct substitution into formula for infinite sum ***A1***

eg $\frac{18}{1-\frac{2\sin^2\theta}{3}}$

evidence of choosing an appropriate rule for $\cos 2\theta$ (seen anywhere) ***(M1)***

eg $\cos 2\theta = 1 - 2\sin^2\theta$

correct substitution of identity/working (seen anywhere) ***(A1)***

eg $\frac{18}{1-\frac{2}{3}\left(\frac{1-\cos 2\theta}{2}\right)}, \frac{54}{3-2\left(\frac{1-\cos 2\theta}{2}\right)}, \frac{18}{\frac{3-2\sin^2\theta}{3}}$

correct working that clearly leads to the given answer ***A1*** *eg* $\frac{18\times3}{2+(1-2\sin^2\theta)}$, $\frac{54}{3-(1-\cos2\theta)}$

$\frac{54}{2+\cos(2\theta)}$ ***AG N0***

[4 marks]

c.

METHOD 1 (using differentiation)

recognizing $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}=0$ (seen anywhere) ***(M1)***

finding any correct expression for $\frac{\mathrm{d}S_\infty}{\mathrm{d}\theta}$ ***(A1)***

eg $\frac{0-54\times(-2\sin2\theta)}{(2+\cos2\theta)^2}$, $-54(2+\cos2\theta)^{-2}\ (-2\sin2\theta)$

correct working ***(A1)***

eg $\sin 2\theta = 0$

any correct value for $\sin^{-1}(0)$ (seen anywhere) ***(A1)***

eg 0, π, … , sketch of sine curve with *x*-intercept(s) marked both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

both correct answers $\theta=\frac{\pi}{2},\frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

METHOD 2 (using denominator)

recognizing when S_∞ is greatest ***(M1)***

eg $2+\cos2\theta$ is a minimum, $1-r$ is smallest
correct working ***(A1)***

eg minimum value of $2+\cos2\theta$ is 1, minimum $r=\frac{2}{3}$

correct working ***(A1)***

eg $\cos2\theta=-1,\ \frac{2}{3}\sin^2\theta=\frac{2}{3},\ \sin^2\theta=1$

EITHER (using $\cos 2\theta$)

any correct value for $\cos^{-1}(-1)$ (seen anywhere) ***(A1)***

eg π, 3π, … (accept values in degrees), sketch of cosine curve with *x*-intercept(s) marked

both correct values for 2θ (ignore additional values) ***(A1)***

$2\theta = \pi, 3\pi$ (accept values in degrees)

OR (using $\sin\theta$)

$\sin\theta = \pm1$ (A1)

$\sin^{-1}(1)=\frac{\pi}{2}$ (accept values in degrees) (seen anywhere) ***A1***

THEN

both correct answers $\theta=\frac{\pi}{2},\frac{3\pi}{2}$ ***A1 N4***

Note: Award ***A0*** if either or both correct answers are given in degrees.
Award ***A0*** if additional values are given.

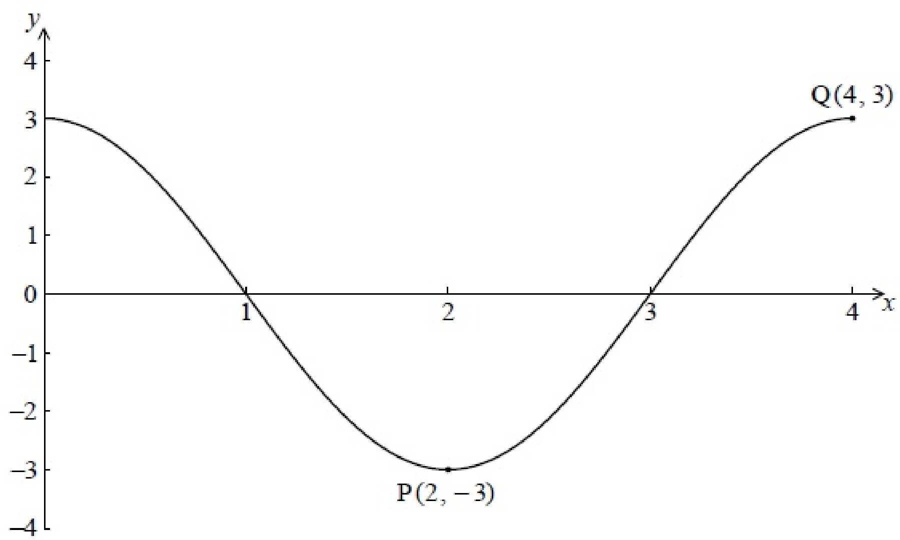
[6 marks]

Examiners report

a.i. [N/A]
[N/A]

b.ii. [N/A]
c. [N/A]

The following diagram shows the graph of $f(x) = a\cos(bx)$, for $0 \le x \le 4$.



There is a minimum point at P(2, − 3) and a maximum point at Q(4, 3) .

a(i) and (ii). (i) Write down the value of a . [3]

(ii) Find the value of b .

b. Write down the gradient of the curve at P. [1]

c. Write down the equation of the normal to the curve at P. [2]

Markscheme

a(i) and (ii). (i) $a = 3$ ***A1 N1***

(ii) **METHOD 1**

attempt to find period ***(M1)***

e.g. 4 , $b = 4$, $\frac{2\pi}{b}$

$b = \frac{2\pi}{4}\left(= \frac{\pi}{2}\right)$ ***A1 N2***

[3 marks]

METHOD 2

attempt to substitute coordinates ***(M1)***

e.g. $3\cos(2b) = -3$, $3\cos(4b) = 3$

$b = \frac{2\pi}{4}\left(= \frac{\pi}{2}\right)$ ***A1 N2***

[3 marks]

b. 0 ***A1 N1***

[1 mark]

c. recognizing that normal is perpendicular to tangent ***(M1)***

e.g. $m_1 \times m_2 = -1$, $m = -\frac{1}{0}$, sketch of vertical line on diagram

$x = 2$ (do not accept 2 or $y = 2$) ***A1*** ***N2***

[2 marks]

Examiners report

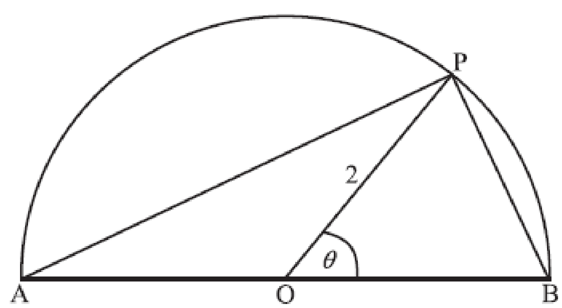
a(i) and (ii). In part (a), many candidates were able to successfully write down the value of *a* as instructed by inspecting the graph and seeing the amplitude of the function is 3. Many also used a formulaic approach to reach the correct answer. When finding the value of *b*, there were many candidates who thought *b* was the period of the function, rather than $\frac{2\pi}{\text{period}}$.

b. In part (b), the directions asked candidates to write down the gradient of the curve at the local minimum point P. However, many candidates spent a good deal of time finding the derivative of the function and finding the value of the derivative for the given value of *x*, rather than simply stating that the gradient of a curve at a minimum point is zero.

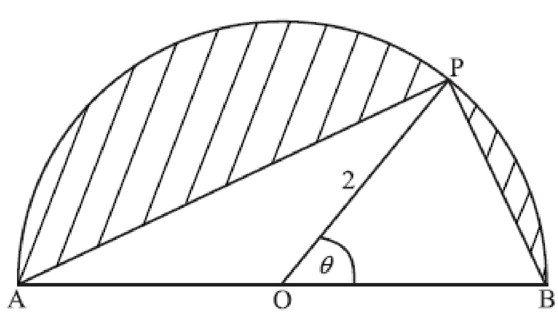
c. For part (c), finding the equation of the normal to the curve, many candidates tried to work with algebraic equations involving negative reciprocal gradients, rather than recognizing that the equation of the vertical line was $x = 2$. There were also candidates who had trouble expressing the correct equation of a line parallel to the *y*-axis.

The following diagram shows a semicircle centre O, diameter [AB], with radius 2.

Let P be a point on the circumference, with $\mathrm{P\widehat{O}B} = \theta$ radians.



Let *S* be the total area of the two segments shaded in the diagram below.



a. Find the area of the triangle OPB, in terms of θ . [2]

b. Explain why the area of triangle OPA is the same as the area triangle OPB. [3]

c. Show that $S = 2(\pi - 2\sin\theta)$. [3]

d. Find the value of θ when S is a local minimum, justifying that it is a minimum. [8]

e. Find a value of θ for which S has its greatest value. [2]

Markscheme

a. evidence of using area of a triangle ***(M1)***

e.g. $A = \frac{1}{2} \times 2 \times 2 \times \sin\theta$

$A = 2\sin\theta$ ***A1*** ***N2***

[2 marks]

b. **METHOD 1**

$\mathrm{P\hat{O}A} = \pi - \theta$ ***(A1)***

area $\Delta\mathrm{OPA} = \frac{1}{2}2 \times 2 \times \sin(\pi - \theta)\ (= 2\sin(\pi - \theta))$ ***A1***

since $\sin(\pi - \theta) = \sin\theta$ ***R1***

then both triangles have the same area ***AG*** ***N0***

METHOD 2

triangle OPA has the same height and the same base as triangle OPB ***R3***

then both triangles have the same area ***AG*** ***N0***

[3 marks]

c. area semicircle $= \frac{1}{2} \times \pi(2)^2\ (= 2\pi)$ ***A1***

area $\Delta\mathrm{APB} = 2\sin\theta + 2\sin\theta\ (= 4\sin\theta)$ ***A1***

$S =$ area of semicircle $-$ area $\Delta\mathrm{APB}$ $(= 2\pi - 4\sin\theta)$ ***M1***

$S = 2(\pi - 2\sin\theta)$ ***AG*** ***N0***

[3 marks]

d. **METHOD 1**

attempt to differentiate ***(M1)***

e.g. $\frac{\mathrm{d}S}{\mathrm{d}\theta} = -4\cos\theta$

setting derivative equal to 0 ***(M1)***

correct equation ***A1***

e.g. $-4\cos\theta = 0$, $\cos\theta = 0$, $4\cos\theta = 0$

$\theta = \frac{\pi}{2}$ ***A1*** ***N3***

EITHER

evidence of using second derivative ***(M1)***

$S''(\theta) = 4\sin\theta$ ***A1***

$S''\left(\frac{\pi}{2}\right) = 4$ ***A1*** it is a minimum because $S''\left(\frac{\pi}{2}\right) > 0$ ***R1*** ***N0***

OR

evidence of using first derivative ***(M1)***

for $\theta < \frac{\pi}{2}$, $S'(\theta) < 0$ (may use diagram) ***A1***

for $\theta > \frac{\pi}{2}$, $S'(\theta) > 0$ (may use diagram) ***A1***

it is a minimum since the derivative goes from negative to positive ***R1*** ***N0***

METHOD 2

$2\pi - 4\sin\theta$ is minimum when $4\sin\theta$ is a maximum ***R3***

$4\sin\theta$ is a maximum when $\sin\theta = 1$ ***(A2)***

$\theta = \frac{\pi}{2}$ ***A3*** ***N3***

[8 marks]

e. S is greatest when $4\sin\theta$ is smallest (or equivalent) ***(R1)***

$\theta = 0$ (or π) ***A1*** ***N2***

[2 marks]

Examiners report

a. Most candidates could obtain the area of triangle OPB as equal to $2\sin\theta$, though 2θ was given quite often as the area.

b. A minority recognized the equality of the sines of supplementary angles and the term complementary was frequently used instead of supplementary. Only a handful of candidates used the simple equal base and altitude argument.

c. Many candidates seemed to see why $S = 2(\pi - 2\sin\theta)$ but the arguments presented for showing why this result was true were not very convincing in many cases. Explicit evidence of why the area of the semicircle was 2π was often missing as was an explanation for $2(2\sin\theta)$ and for subtraction.

d. Only a small number of candidates recognized the fact S would be minimum when sin was maximum, leading to a simple non-calculus solution. Those who chose the calculus route often had difficulty finding the derivative of S, failing in a significant number of cases to recognize that the derivative of a constant is 0, and also going through painstaking application of the product rule to find the simple derivative. When it came to justify a minimum, there was evidence in some cases of using some form of valid test, but explanation of the test being used was generally poor.

e. Candidates who answered part (d) correctly generally did well in part (e) as well, though answers outside the domain of θ were frequently seen.

Let $f(x) = 6x\sqrt{1-x^2}$, for $-1 \leqslant x \leqslant 1$, and $g(x) = \cos(x)$, for $0 \leqslant x \leqslant \pi$.

Let $h(x) = (f \circ g)(x)$.

a. Write $h(x)$ in the form $a\sin(bx)$, where $a,\ b \in \mathbb{Z}$. [5]

b. Hence find the range of h. [2]

Markscheme

a. attempt to form composite in any order ***(M1)***

eg $f\left(g(x)\right),\ \cos\left(6x\sqrt{1-x^2}\right)$

correct working ***(A1)***

eg $6\cos x\sqrt{1-\cos^2 x}$

correct application of Pythagorean identity (do not accept $\sin^2 x+\cos^2 x=1$) ***(A1)***

eg $\sin^2 x=1-\cos^2 x,\ 6\cos x\sin x,\ 6\cos x\left|\sin x\right|$

valid approach (do not accept $2\sin x\cos x=\sin 2x$) ***(M1)***

eg $3(2\cos x\sin x)$

$h(x)=3\sin 2x$ ***A1 N3***

[5 marks]

b. valid approach ***(M1)***

eg amplitude $=3$, sketch with max and min y-values labelled, $-3<y<3$

correct range ***A1 N2***

eg $-3\leqslant y\leqslant 3$, $[-3,\ 3]$ from -3 to 3

Note: Do not award ***A1*** for $-3<y<3$ or for "between -3 and 3".

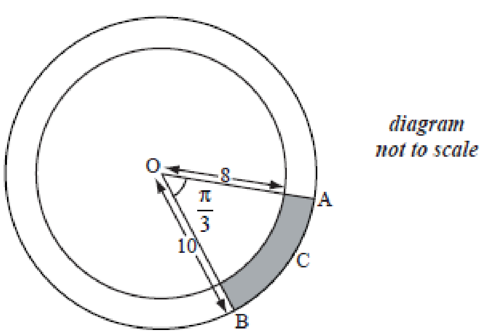
[2 marks]

Examiners report

a. In part (a), nearly all candidates found the correct composite function in terms of $\cos x$, though many did not get any further than this first step in their solution to the question. While some candidates seemed to recognize the need to use trigonometric identities, most were unsuccessful in finding the correct expression in the required form.

b. In part (b), very few candidates were able to provide the correct range of the function.

The diagram shows two concentric circles with centre O.



The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm.

Points A, B and C are on the circumference of the larger circle such that $\text{A}\widehat{\text{O}}\text{B}$ is $\frac{\pi}{3}$ radians.

a. Find the length of the arc ACB . [2]

b. Find the area of the shaded region. [4]

Markscheme

a. correct substitution in $l = r\theta$ ***(A1)***

e.g. $10 \times \frac{\pi}{3}$, $\frac{1}{6} \times 2\pi \times 10$

arc length $= \frac{20\pi}{6} \left(= \frac{10\pi}{3}\right)$ ***A1 N2***

[2 marks]

b. area of large sector $= \frac{1}{2} \times 10^2 \times \frac{\pi}{3} \left(= \frac{100\pi}{6}\right)$ ***(A1)***

area of small sector $= \frac{1}{2} \times 8^2 \times \frac{\pi}{3} \left(= \frac{64\pi}{6}\right)$ ***(A1)***

evidence of valid approach (seen anywhere) ***M1***

e.g. subtracting areas of two sectors, $\frac{1}{2} \times \frac{\pi}{3}(10^2 - 8^2)$

area shaded $= 6\pi$ (accept $\frac{36\pi}{6}$, etc.) ***A1 N3***

[4 marks]

Examiners report

a. This question was very well done by the majority of candidates. Some candidates correctly substituted the values into the formulas, but failed to do the calculations and write their answers in finished form.

b. Nearly all used the correct method of subtracting the sector areas in part (b), though multiplying with fractions proved challenging for some candidates.

The vertices of the triangle PQR are defined by the position vectors

$$\overrightarrow{\mathrm{OP}} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}, \overrightarrow{\mathrm{OQ}} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \text{ and } \overrightarrow{\mathrm{OR}} = \begin{pmatrix} 6 \\ -1 \\ 5 \end{pmatrix}.$$

a. Find [3]

(i) $\overrightarrow{\mathrm{PQ}}$;

(ii) $\overrightarrow{\mathrm{PR}}$.

b. Show that $\cos \mathrm{R\widehat{P}Q} = \frac{1}{2}$. [7]

c. (i) Find $\sin\mathrm{R\widehat{P}Q}$. [6]

(ii) Hence, find the area of triangle PQR, giving your answer in the form $a\sqrt{3}$.

Markscheme

a. (i) evidence of approach ***(M1)***

e.g. $\overrightarrow{\mathrm{PQ}} = \overrightarrow{\mathrm{PO}} + \overrightarrow{\mathrm{OQ}}$, $\mathrm{Q} - \mathrm{P}$

$\overrightarrow{\mathrm{PQ}} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$ ***A1 N2***

(ii) $\overrightarrow{\mathrm{PR}} = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}$ ***A1 N1***

[3 marks]

b. **METHOD 1**

choosing correct vectors $\overrightarrow{\mathrm{PQ}}$ and $\overrightarrow{\mathrm{PR}}$ ***(A1)(A1)***

finding $\overrightarrow{\mathrm{PQ}} \bullet \overrightarrow{\mathrm{PR}}$, $\left|\overrightarrow{\mathrm{PQ}}\right|$, $\left|\overrightarrow{\mathrm{PR}}\right|$ ***(A1) (A1)(A1)***

$\overrightarrow{\mathrm{PQ}} \bullet \overrightarrow{\mathrm{PR}} = -2 + 4 + 4 (= 6)$

$\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{(-1)^2 + 2^2 + 1^2} \left(= \sqrt{6}\right)$, $\left|\overrightarrow{\mathrm{PR}}\right| = \sqrt{2^2 + 2^2 + 4^2} \left(= \sqrt{24}\right)$

substituting into formula for angle between two vectors ***M1***

e.g. $\cos \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \frac{6}{\sqrt{6} \times \sqrt{24}}$

simplifying to expression clearly leading to $\frac{1}{2}$ ***A1***

e.g. $\frac{6}{\sqrt{6} \times 2\sqrt{6}}$, $\frac{6}{\sqrt{144}}$, $\frac{6}{12}$

$\cos \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \frac{1}{2}$ ***AG N0***

METHOD 2

evidence of choosing cosine rule (seen anywhere) ***(M1)***

$\overrightarrow{\mathrm{QR}} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix}$ ***A1***

$\left|\overrightarrow{\mathrm{QR}}\right| = \sqrt{18}$, $\left|\overrightarrow{\mathrm{PQ}}\right| = \sqrt{6}$ and $\left|\overrightarrow{\mathrm{PR}}\right| = \sqrt{24}$ ***(A1)(A1)(A1)***

$\cos \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \frac{\left(\sqrt{6}\right)^2 + \left(\sqrt{24}\right)^2 - \left(\sqrt{18}\right)^2}{2\sqrt{6} \times \sqrt{24}}$ ***A1***

$\cos \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \frac{6 + 24 - 18}{24} \left(= \frac{12}{24}\right)$ ***A1***

$\cos \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \frac{1}{2}$ ***AG N0***

[7 marks]

c. (i) **METHOD 1**

evidence of appropriate approach ***(M1)***

e.g. using $\sin^2 \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} + \cos^2 \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = 1$, diagram

substituting correctly ***(A1)***

e.g. $\sin \mathrm{R}\widehat{\mathrm{P}}\mathrm{Q} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$

$\sin R\widehat{P}Q = \sqrt{\frac{3}{4}} \left(= \frac{\sqrt{3}}{2}\right)$ ***A1*** ***N3***

METHOD 2

since $\cos \widehat{P} = \frac{1}{2}$, $\widehat{P} = 60°$ ***(A1)***

evidence of approach

e.g. drawing a right triangle, finding the missing side ***(A1)***

$\sin \widehat{P} = \frac{\sqrt{3}}{2}$ ***A1*** ***N3***

(ii) evidence of appropriate approach ***(M1)***

e.g. attempt to substitute into $\frac{1}{2}ab \sin C$

correct substitution

e.g. area $= \frac{1}{2}\sqrt{6} \times \sqrt{24} \times \frac{\sqrt{3}}{2}$ ***A1***

area $= 3\sqrt{3}$ ***A1*** ***N2***

[6 marks]

Examiners report

a. Combining the vectors in (a) was generally well done, although some candidates reversed the subtraction, while others calculated the magnitudes.

b. Many candidates successfully used scalar product and magnitude calculations to complete part (b). Alternatively, some used the cosine rule, and often achieved correct results. Some assumed the triangle was a right-angled triangle and thus did not earn full marks. Although PQR is indeed right-angled, in a “show that” question this attribute must be directly established.

c. Many candidates attained the value for sine in (c) with little difficulty, some using the Pythagorean identity, while others knew the side relationships in a 30-60-90 triangle. Unfortunately, a good number of candidates then used the side values of $1, 2, \sqrt{3}$ to find the area of PQR , instead of the magnitudes of the vectors found in (a). Furthermore, the "hence" command was sometimes neglected as the value of sine was expected to be used in the approach.

Solve the equation $2 \cos x = \sin 2x$, for $0 \le x \le 3\pi$.

Markscheme

METHOD 1

using double-angle identity (seen anywhere) ***A1***

e.g. $\sin 2x = 2 \sin x \cos x$, $2 \cos x = 2 \sin x \cos x$

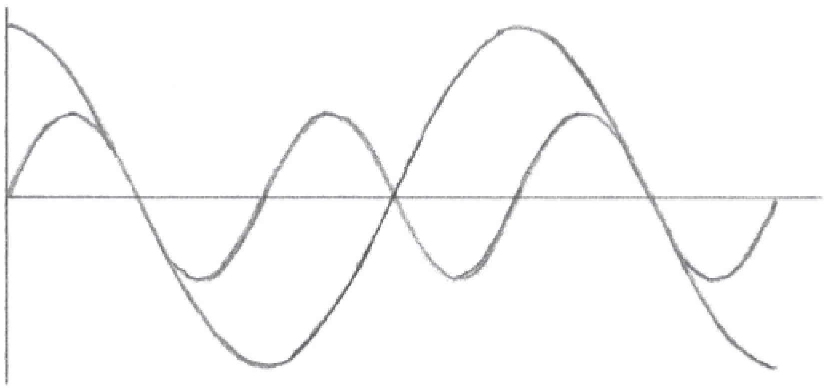
evidence of valid attempt to solve equation ***(M1)***

e.g. $0 = 2 \sin x \cos x - 2 \cos x$, $2 \cos x(1 - \sin x) = 0$

$\cos x = 0$, $\sin x = 1$ ***A1A1***

$x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$ ***A1A1A1*** ***N4***

METHOD 2



A1A1M1A1

Notes: Award ***A1*** for sketch of $\sin 2x$, ***A1*** for a sketch of $2\cos x$, ***M1*** for at least one intersection point seen, and ***A1*** for 3 approximately correct intersection points. Accept sketches drawn outside $[0, 3\pi]$, even those with more than 3 intersections.

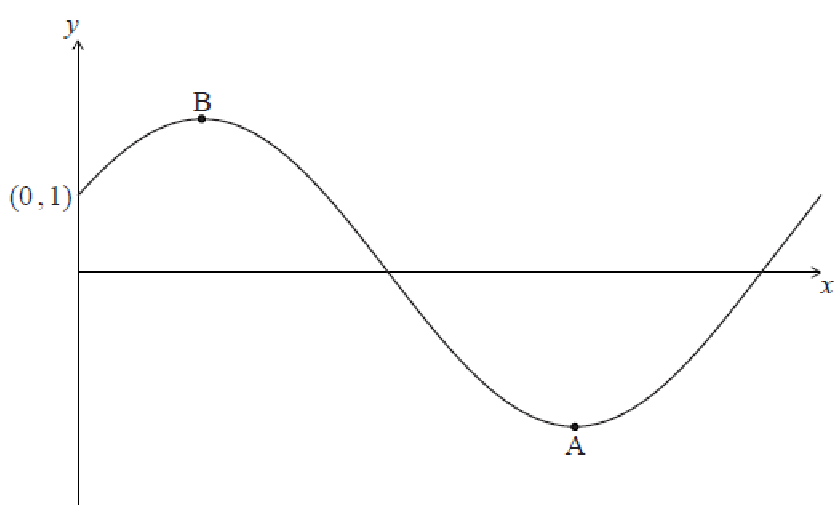
$x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$, $x = \frac{5\pi}{2}$ ***A1A1A1*** ***N4***

[7 marks]

Examiners report

By far the most common error was to "cancel" the $\cos x$ and find only two of the three solutions. It was disappointing how few candidates solved this by setting factors equal to zero. Some candidates wrote all three answers from $\sin x = 1$, which only earned two of the three final marks. On a brighter note, many candidates found the $\frac{5\pi}{2}$, which showed an appreciation for the period of the function as well as the domain restriction. A handful of candidates cleverly sketched both graphs and used the intersections to find the three solutions.

Let $f(x) = \cos x + \sqrt{3} \sin x$, $0 \le x \le 2\pi$. The following diagram shows the graph of f .



The y-intercept is at (0, 1) , there is a minimum point at A (p, q) and a maximum point at B.

a. Find $f'(x)$. [2]

b(i) and (ii). [10]

(i) show that $q = -2$;

(ii) verify that A is a minimum point.

c. Find the maximum value of $f(x)$. [3]

d. The function $f(x)$ can be written in the form $r\cos(x-a)$. [2]

Write down the value of r and of a .

Markscheme

a. $f'(x) = -\sin x + \sqrt{3}\cos x$ ***A1A1 N2***

[2 marks]

b(i) and (ii). (i) at A, $f'(x) = 0$ ***R1***

correct working ***A1***

e.g. $\sin x = \sqrt{3}\cos x$

$\tan x = \sqrt{3}$ ***A1***

$x = \frac{\pi}{3}, \frac{4\pi}{3}$ ***A1***

attempt to substitute **their** x into $f(x)$ ***M1***

e.g. $\cos\left(\frac{4\pi}{3}\right) + \sqrt{3}\sin\left(\frac{4\pi}{3}\right)$

correct substitution ***A1***

e.g. $-\frac{1}{2} + \sqrt{3}\left(-\frac{\sqrt{3}}{2}\right)$

correct working that clearly leads to -2 ***A1***

e.g. $-\frac{1}{2} - \frac{3}{2}$

$q = -2$ ***AG N0***

(ii) correct calculations to find $f'(x)$ either side of $x = \frac{4\pi}{3}$ ***A1A1***

e.g. $f'(\pi) = 0 - \sqrt{3}$, $f'(2\pi) = 0 + \sqrt{3}$

$f'(x)$ changes sign from negative to positive ***R1***

so A is a minimum ***AG N0***

[10 marks]

c. max when $x = \frac{\pi}{3}$ ***R1***

correctly substituting $x = \frac{\pi}{3}$ into $f(x)$ ***A1***

e.g. $\frac{1}{2} + \sqrt{3}\left(\frac{\sqrt{3}}{2}\right)$

max value is 2 ***A1 N1***

[3 marks]

d. $r = 2$, $a = \frac{\pi}{3}$ ***A1A1 N2***

[2 marks]

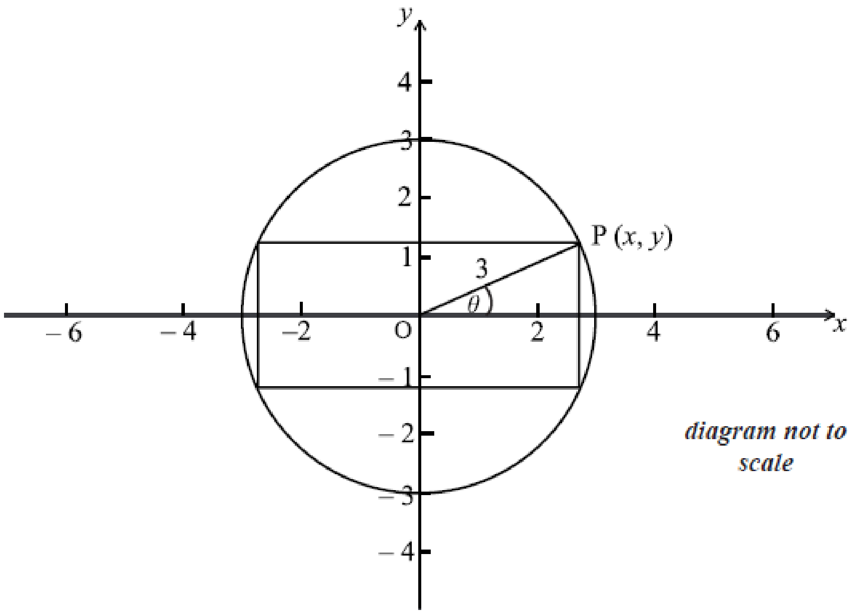
Examiners report

a. [N/A]
[N/A]

b(i) and (ii). [N/A]

d. [N/A]

A rectangle is inscribed in a circle of radius 3 cm and centre O, as shown below.



The point P(x , y) is a vertex of the rectangle and also lies on the circle. The angle between (OP) and the x-axis is θ radians, where $0 \le \theta \le \frac{\pi}{2}$.

a. Write down an expression in terms of θ for [2]

(i) x ;

(ii) y .

b. Let the area of the rectangle be A. [3]

Show that $A = 18\sin 2\theta$.

c. (i) Find $\frac{\mathrm{d}A}{\mathrm{d}\theta}$. [8]

(ii) Hence, find the exact value of θ which maximizes the area of the rectangle.

(iii) Use the second derivative to justify that this value of θ does give a maximum.

Markscheme

a. (i) $x = 3\cos\theta$ ***A1*** ***N1***

(ii) $y = 3\sin\theta$ ***A1*** ***N1***

[2 marks]

b. finding area ***(M1)***

e.g. $A = 2x \times 2y$, $A = 8 \times \frac{1}{2}bh$

substituting ***A1***

e.g. $A = 4 \times 3\sin\theta \times 3\cos\theta$, $8 \times \frac{1}{2} \times 3\cos\theta \times 3\sin\theta$

$A = 18(2\sin\theta\cos\theta)$ ***A1***

$A = 18\sin 2\theta$ ***AG*** ***N0***

[3 marks]

c. (i) $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 36\cos 2\theta$ ***A2*** ***N2***

(ii) for setting derivative equal to 0 ***(M1)***

e.g. $36\cos 2\theta = 0$, $\frac{\mathrm{d}A}{\mathrm{d}\theta} = 0$

$2\theta = \frac{\pi}{2}$ ***(A1)***

$\theta = \frac{\pi}{4}$ ***A1*** ***N2***

(iii) valid reason (seen anywhere) ***R1***

e.g. at $\frac{\pi}{4}$, $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} < 0$; maximum when $f''(x) < 0$

finding second derivative $\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2} = -72\sin 2\theta$ ***A1***

evidence of substituting $\frac{\pi}{4}$ ***M1***

e.g. $-72\sin\left(2\times\frac{\pi}{4}\right)$, $-72\sin\left(\frac{\pi}{2}\right)$, -72

$\theta = \frac{\pi}{4}$ produces the maximum area ***AG*** ***N0***

[8 marks]

Examiners report

a. Candidates familiar with the circular nature of sine and cosine found part (a) accessible. However, a good number of candidates left this part blank, which suggests that there was difficulty interpreting the meaning of the *x* and *y* in the diagram.

b. Those with answers from (a) could begin part (b), but many worked backwards and thus earned no marks. In a "show that" question, a solution cannot begin with the answer given. The area of the rectangle could be found by using $2x \times 2y$, or by using the eight small triangles, but it was essential that the substitution of the double-angle formula was shown before writing the given answer.

c. As the area function was given in part (b), many candidates correctly found the derivative in (c) and knew to set this derivative to zero for a maximum value. Many gave answers in degrees, however, despite the given domain in radians.

Although some candidates found the second derivative function correctly, few stated that the second derivative must be negative at a maximum value. Simply calculating a negative value is not sufficient for a justification.

Let $h(x) = \frac{6x}{\cos x}$. Find $h'(0)$.

Markscheme

METHOD 1 (quotient)

derivative of numerator is 6 ***(A1)***

derivative of denominator is $-\sin x$ ***(A1)***

attempt to substitute into quotient rule ***(M1)***

correct substitution ***A1***

e.g. $\frac{(\cos x)(6)-(6x)(-\sin x)}{(\cos x)^2}$

substituting $x = 0$ ***(A1)***

e.g. $\frac{(\cos 0)(6)-(6\times 0)(-\sin 0)}{(\cos 0)^2}$

$h'(0) = 6$ ***A1 N2***

METHOD 2 (product)

$h(x) = 6x \times (\cos x)^{-1}$

derivative of $6x$ is 6 ***(A1)***

derivative of $(\cos x)^{-1}$ is $(-(\cos x)^{-2}(-\sin x))$ ***(A1)***

attempt to substitute into product rule **(M1)**

correct substitution ***A1***

e.g. $(6x)(-(\cos x)^{-2}(-\sin x)) + (6)(\cos x)^{-1}$

substituting $x = 0$ ***(A1)***

e.g. $(6 \times 0)(-(\cos 0)^{-2}(-\sin 0)) + (6)(\cos 0)^{-1}$

$h'(0) = 6$ ***A1 N2***

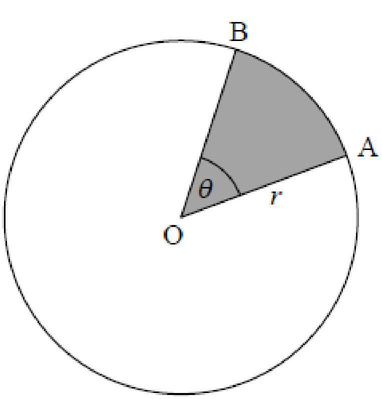
[6 marks]

Examiners report

The majority of candidates were successful in using the quotient rule, and were able to earn most of the marks for this question. However, there were a large number of candidates who substituted correctly into the quotient rule, but then went on to make mistakes in simplifying this expression. These algebraic errors kept the candidates from earning the final mark for the correct answer. A few candidates tried to use the product rule to find the derivative, but they were generally not as successful as those who used the quotient rule. It was pleasing to note that most candidates did know the correct values for the sine and cosine of zero.

The following diagram shows a circle with centre O and radius *r* cm.

diagram not to scale



The points A and B lie on the circumference of the circle, and $\mathrm{A}\hat{\mathrm{O}}\mathrm{B} = \theta$. The area of the shaded sector AOB is 12 cm^2 and the length of arc AB is 6 cm.

Find the value of *r*.

Markscheme

evidence of correctly substituting into circle formula (may be seen later) ***A1A1***

eg $\frac{1}{2}\theta r^2 = 12,\ r\theta = 6$

attempt to eliminate one variable ***(M1)***

eg $r = \frac{6}{\theta},\ \theta = \frac{1}{r},\ \frac{\frac{1}{2}\theta r^2}{r\theta} = \frac{12}{6}$

correct elimination ***(A1)***

eg $\frac{1}{2} \times \frac{6}{r} \times r^2 = 12,\ \frac{1}{2}\theta \times \left(\frac{6}{\theta}\right)^2 = 12,\ A = \frac{1}{2} \times r^2 \times \frac{l}{r},\ \frac{r^2}{2r} = 2$

correct equation ***(A1)***

eg $\frac{1}{2} \times 6r = 12,\ \frac{1}{2} \times \frac{36}{\theta} = 12,\ 12 = \frac{1}{2} \times r^2 \times \frac{6}{r}$

correct working ***(A1)***

eg $3r = 12,\ \frac{18}{\theta} = 12,\ \frac{r}{2} = 2,\ 24 = 6r$

r = 4 (cm) ***A1 N2***

[7 marks]

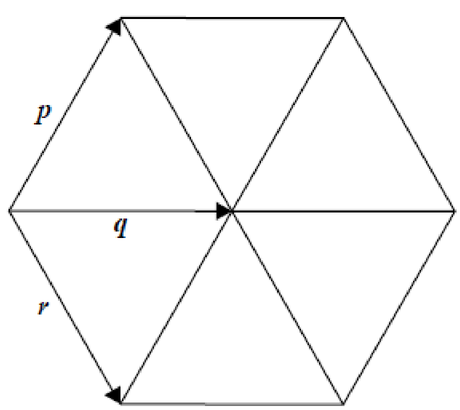
Examiners report

[N/A]

Six equilateral triangles, each with side length 3 cm, are arranged to form a hexagon.

This is shown in the following diagram.

diagram not to scale

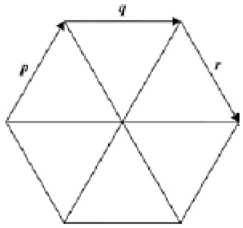


The vectors ***p*** , ***q*** and ***r*** are shown on the diagram.

Find ***p***•(***p*** + ***q*** + ***r***).

Markscheme

METHOD 1 (using |***p***| |2***q***| cos*θ*)

finding ***p*** + ***q*** + ***r*** ***(A1)*** *eg* 2***q***, 

| ***p*** + ***q*** + ***r*** | = 2 × 3 (= 6) (seen anywhere) ***A1***

correct angle between ***p*** and ***q*** (seen anywhere) ***(A1)***

$\frac{\pi}{3}$ (accept 60°)

substitution of **their** values ***(M1)***

eg 3 × 6 × cos$\left(\frac{\pi}{3}\right)$

correct value for cos$\left(\frac{\pi}{3}\right)$ (seen anywhere) ***(A1)***

eg $\frac{1}{2},\ 3\times 6\times\frac{1}{2}$

p•(***p*** + ***q*** + ***r***) = 9 ***A1 N3***

METHOD 2 (scalar product using distributive law)

correct expression for scalar distribution ***(A1)***

eg ***p***• ***p*** + ***p***•***q*** + ***p***•***r***

three correct angles between the vector pairs (seen anywhere) ***(A2)***

eg 0° between ***p*** and ***p***, $\frac{\pi}{3}$ between ***p*** and ***q***, $\frac{2\pi}{3}$ between ***p*** and ***r***

Note: Award ***A1*** for only two correct angles.

substitution of **their** values ***(M1)***

eg 3.3.cos0 +3.3.cos$\frac{\pi}{3}$ + 3.3.cos120

one correct value for cos0, cos$\left(\frac{\pi}{3}\right)$ or cos$\left(\frac{2\pi}{3}\right)$ (seen anywhere) ***A1***

eg $\frac{1}{2},\ 3\times 6\times\frac{1}{2}$

p•(***p*** + ***q*** + ***r***) = 9 ***A1 N3***

METHOD 3 (scalar product using relative position vectors)

valid attempt to find one component of ***p*** or ***r*** ***(M1)***

eg sin 60 = $\frac{x}{3}$, cos 60 = $\frac{x}{3}$, one correct value $\frac{3}{2},\ \frac{3\sqrt{3}}{2},\ \frac{-3\sqrt{3}}{2}$

one correct vector (two or three dimensions) (seen anywhere) ***A1***

$$eg\ \ p=\begin{pmatrix}\dfrac{3}{2}\\ \dfrac{3\sqrt{3}}{2}\end{pmatrix},\ q=\begin{pmatrix}3\\0\end{pmatrix},\ r=\begin{pmatrix}\dfrac{3}{2}\\ -\dfrac{3\sqrt{3}}{2}\\ 0\end{pmatrix}$$

three correct vectors ***p*** + ***q*** + ***r*** = 2***q*** ***(A1)***

p + ***q*** + ***r*** = $\begin{pmatrix}6\\0\end{pmatrix}$ or $\begin{pmatrix}6\\0\\0\end{pmatrix}$ (seen anywhere, including scalar product) ***(A1)***

correct working ***(A1)***

eg $\left(\frac{3}{2}\times 6\right)+\left(\frac{3\sqrt{3}}{2}\times 0\right),\ 9+0+0$

p•(***p*** + ***q*** + ***r***) = 9 ***A1 N3***

[6 marks]

Examiners report

[N/A]